Pairing-Based Batch Arguments for NP with a Linear-Size CRS

Binyi Chen — Stanford University

Noel Elias — UT-Austin

David Wu — UT-Austin

Batch Arguments for NP

<u>Boolean circuit satisfiability</u>

$$\mathcal{L}_C = \{x \in \{0,1\}^n : \exists w, C(x,w) = 1\}$$

Prover

$(x_1, \ldots, x_\ell)$ →

<u>Goal:</u> convince verifier that $x_i \in \mathcal{L}_C$ for all $i \in [\ell]$

Verifier

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Similar for verifier time
(beyond reading statements)

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Different Paths towards BARGs

SNARGs:

- iO or knowledge assumptions
- Or rely on the Random Oracle Model

Correlation Intractability:

[CJJ'21a, CJJ'21b…]

- CI-hash is a heavy machinery

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[WW'22…]

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Example Parameters:

- CRS for $\ell = 10^5$: $> 10^8$ group elements
- Recursion? [WW'22] : Non-black-box crypto + Impractical

Q: Pairing-based BARG with linear-size CRS & quasi-linear prover time?

Our Results

A New Pairing-based BARG for NP

- CRS size: Linear in the # of instances ℓ
- Prover time: $\approx \tilde{O}_{\lambda}(|C| \cdot \ell)$
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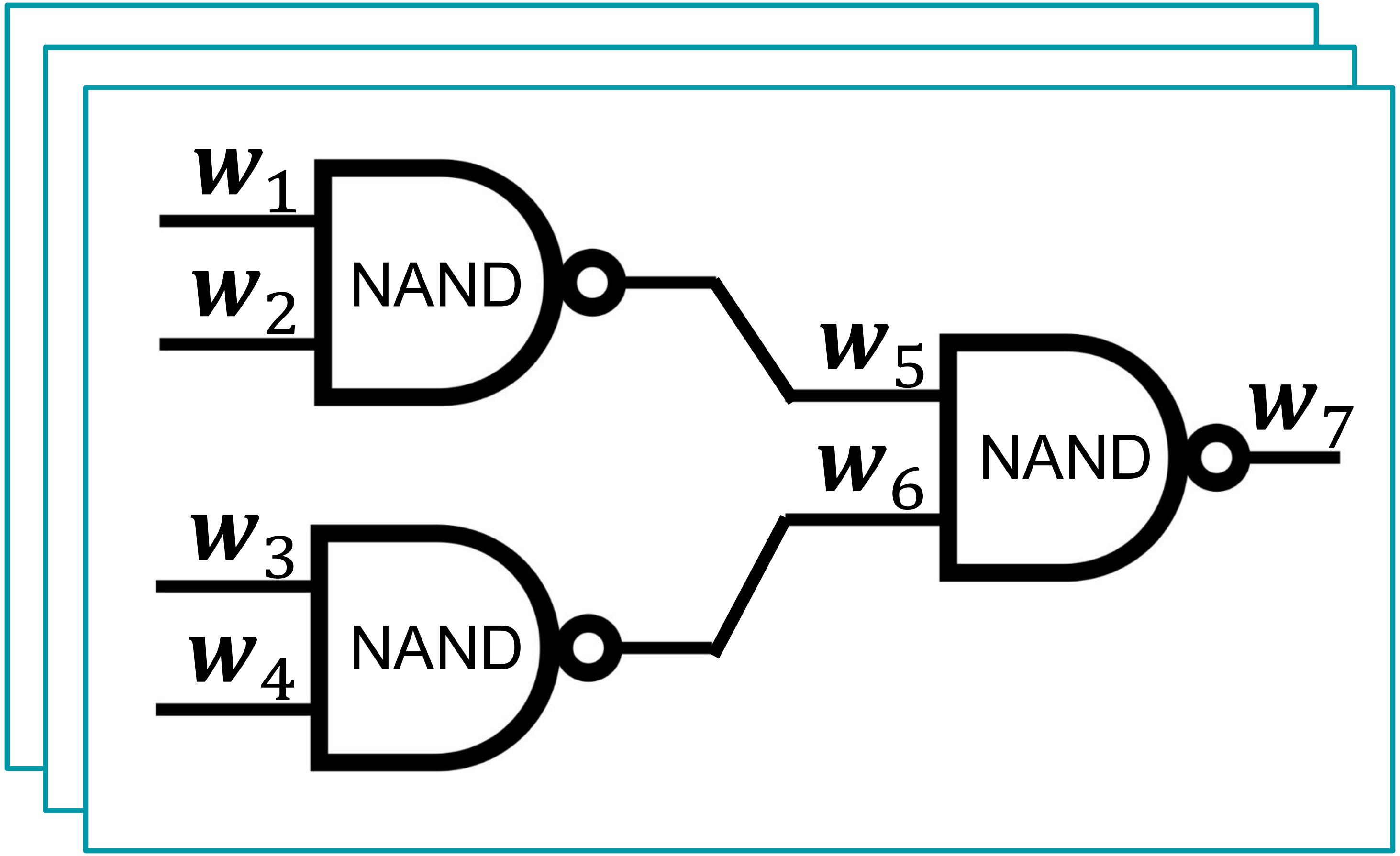
Proven secure in the GGM

Commit-and-Prove for BARG [Waters, Wu, Crypto'22]



Vector of labels for wire i across ℓ instances

$$\boldsymbol{w}_i = (w_{i,1}, w_{i,2}, \dots, w_{i,\ell})$$

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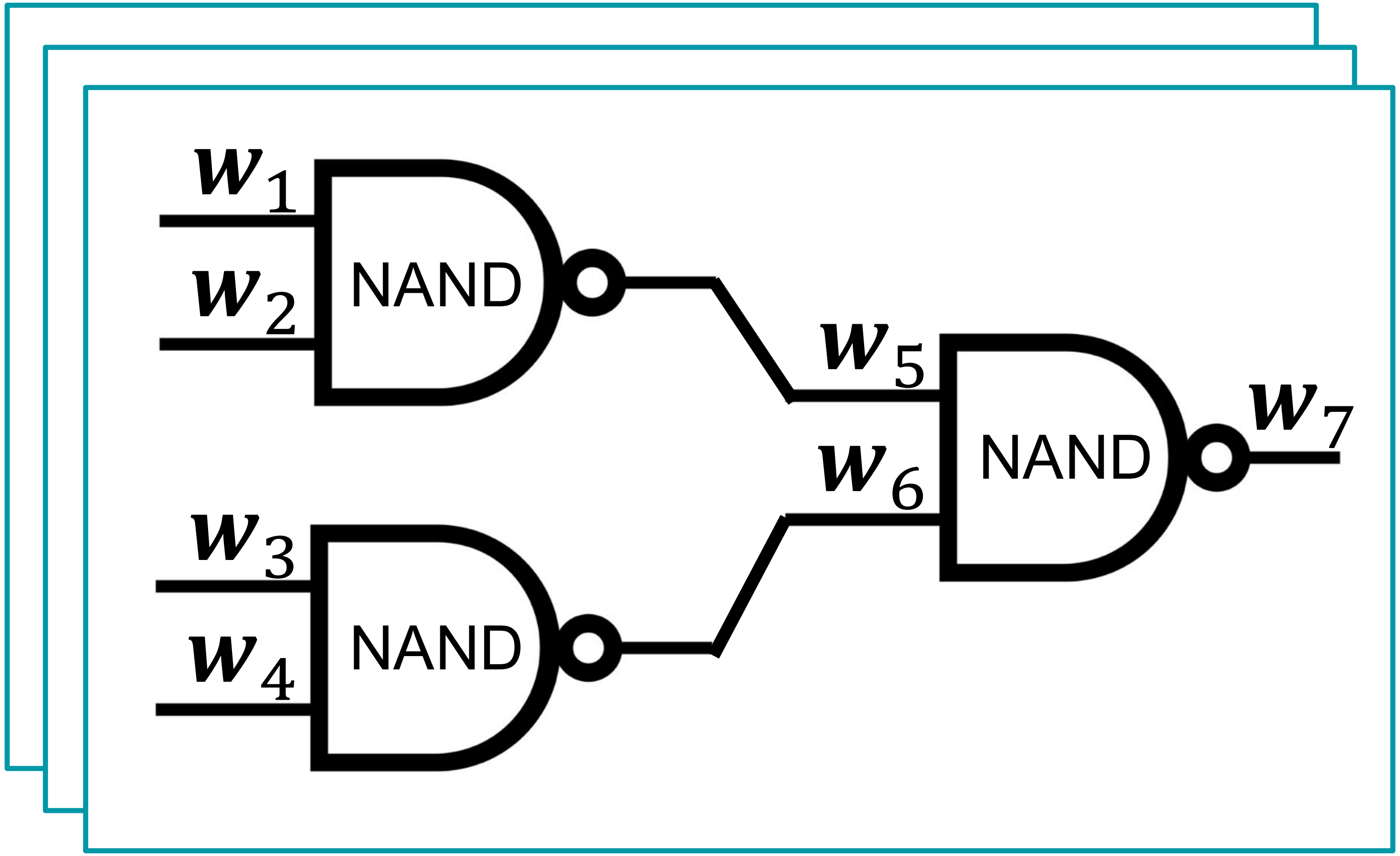

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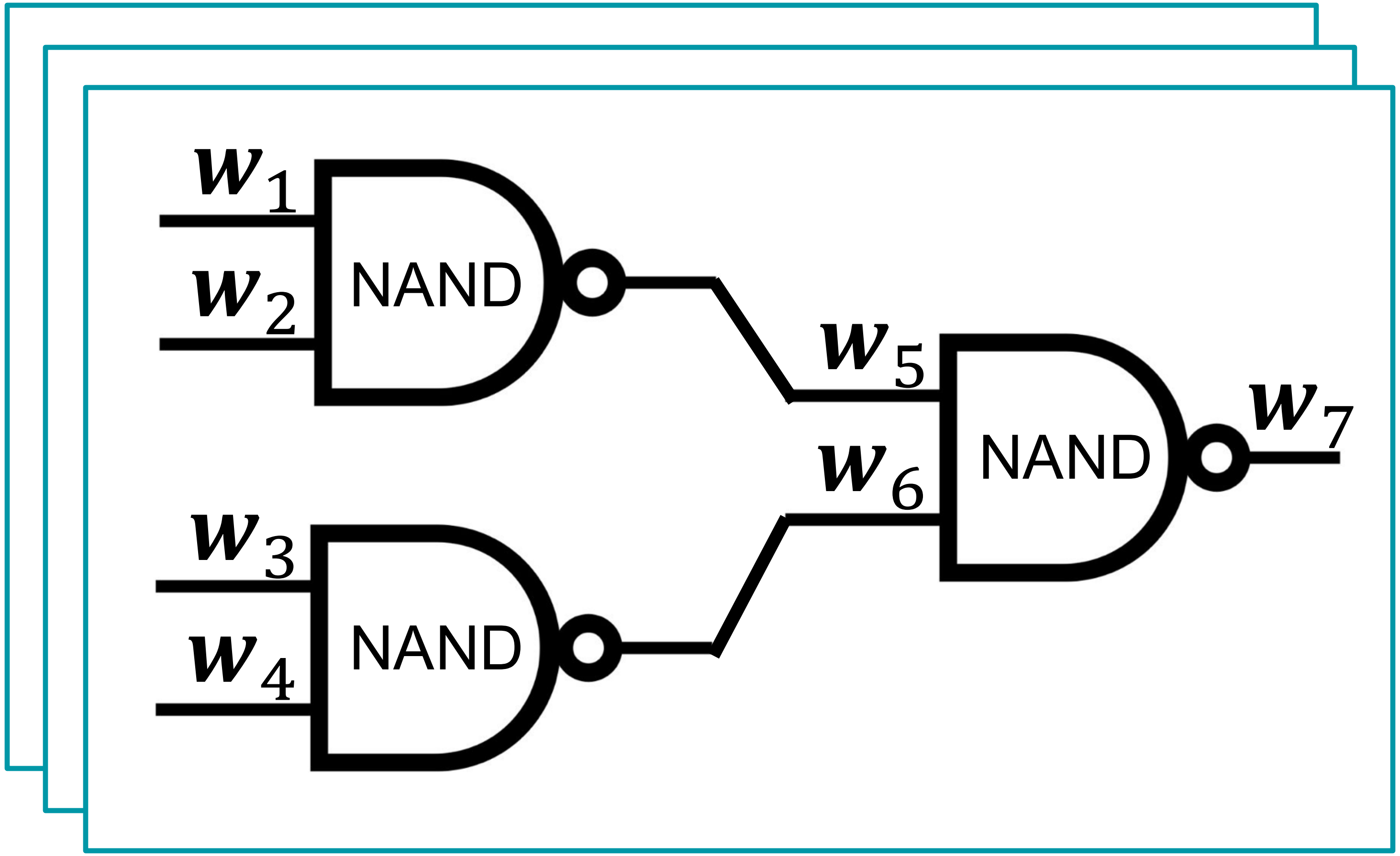

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Validity proofs

Wire validity

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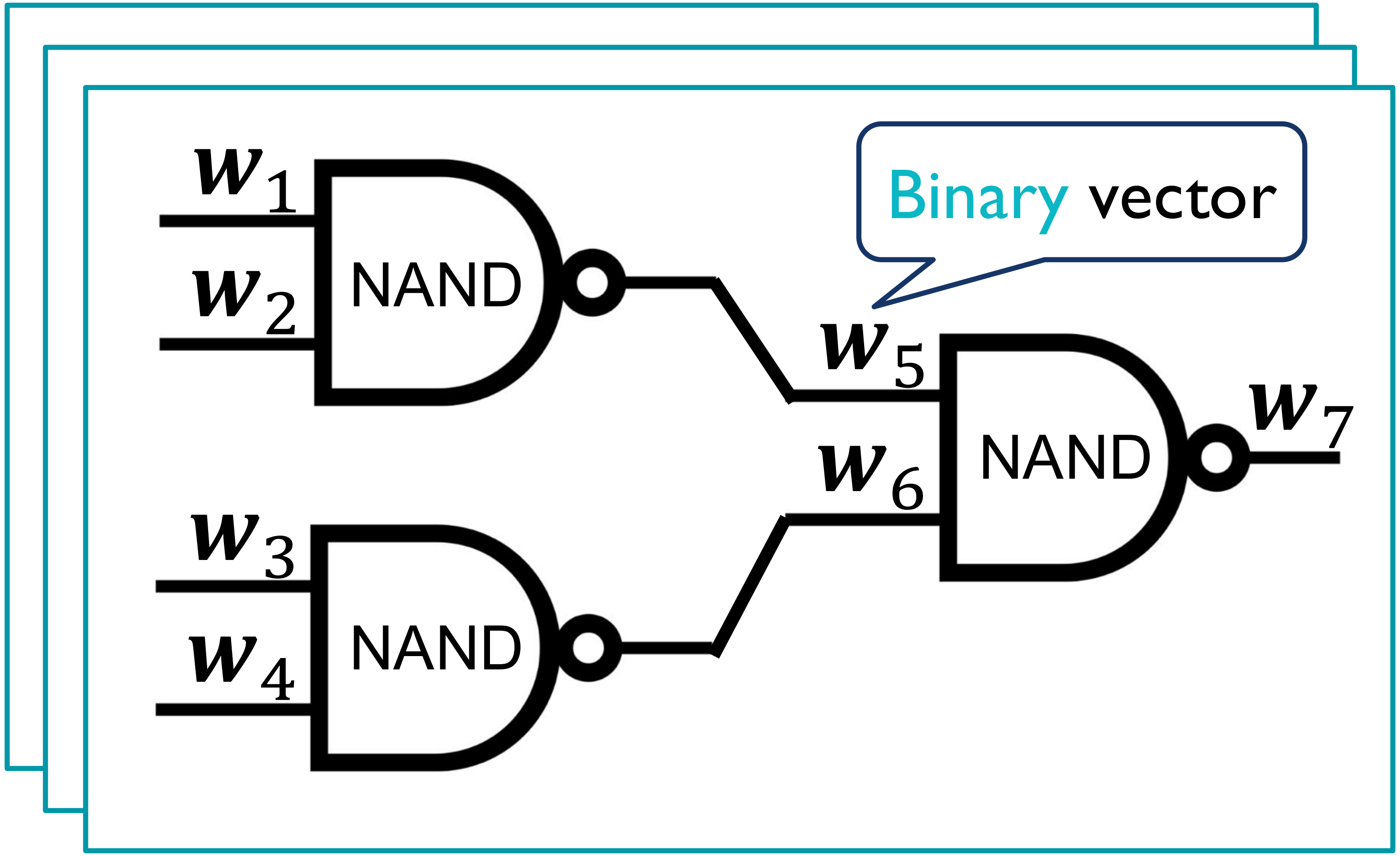

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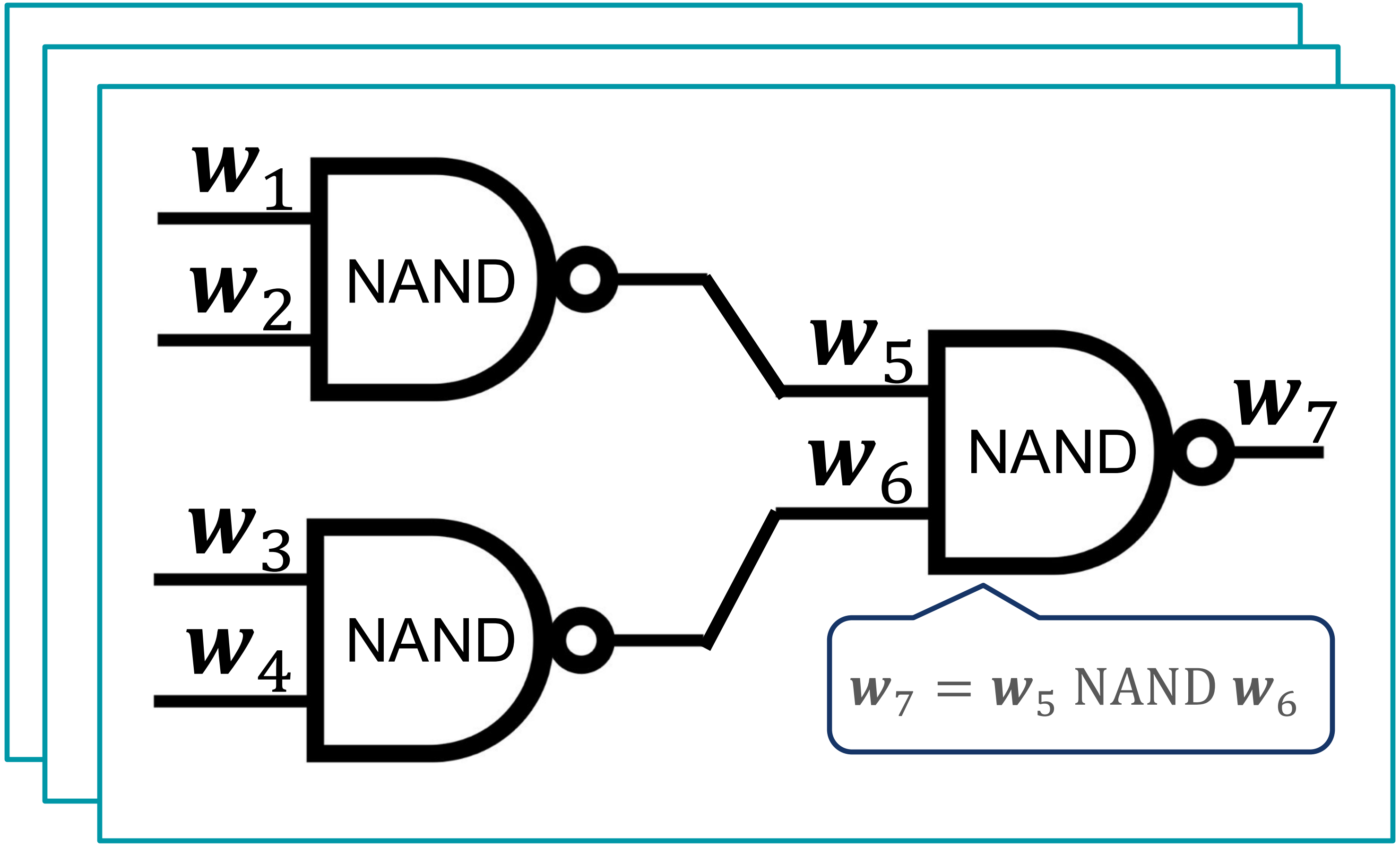

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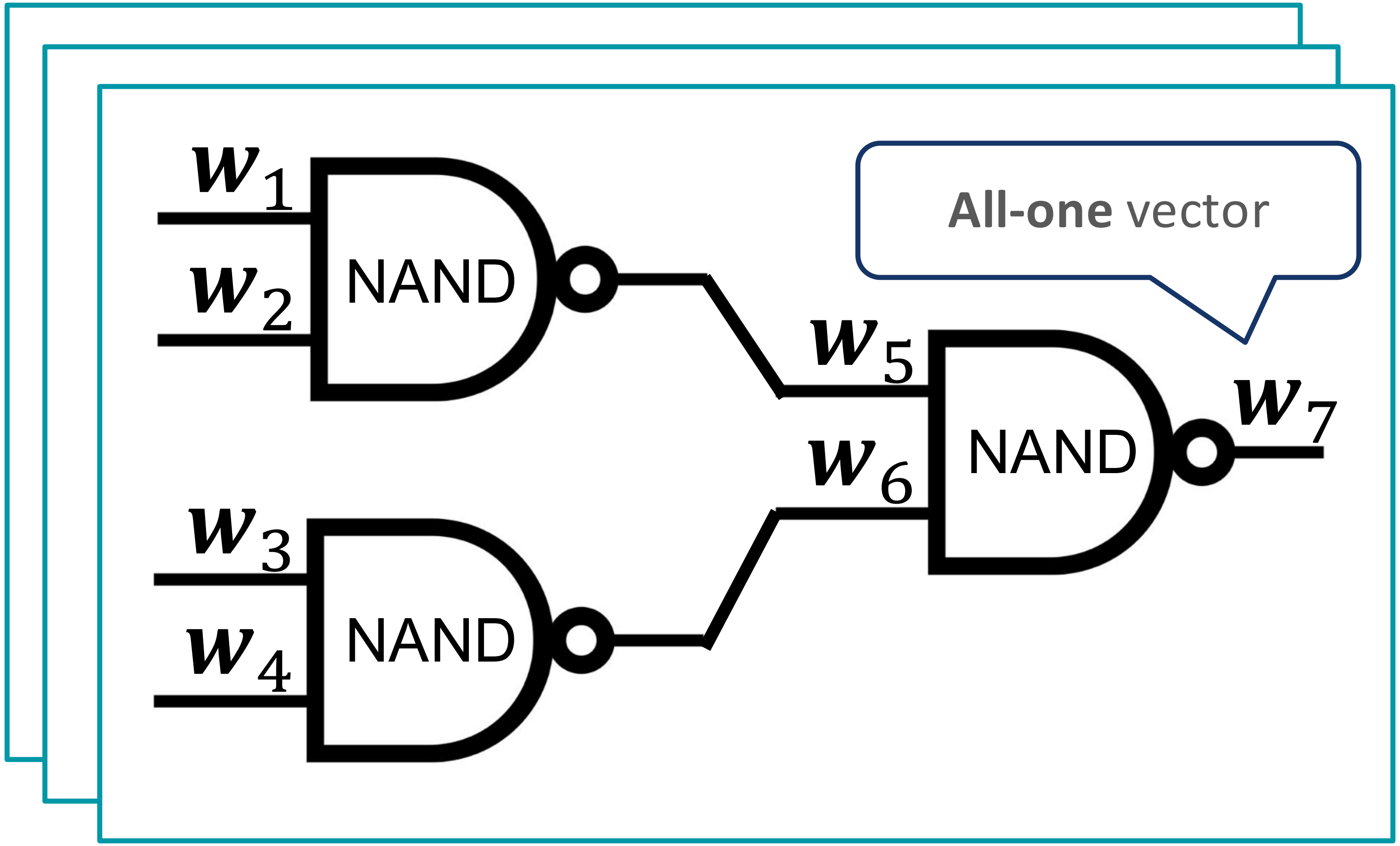

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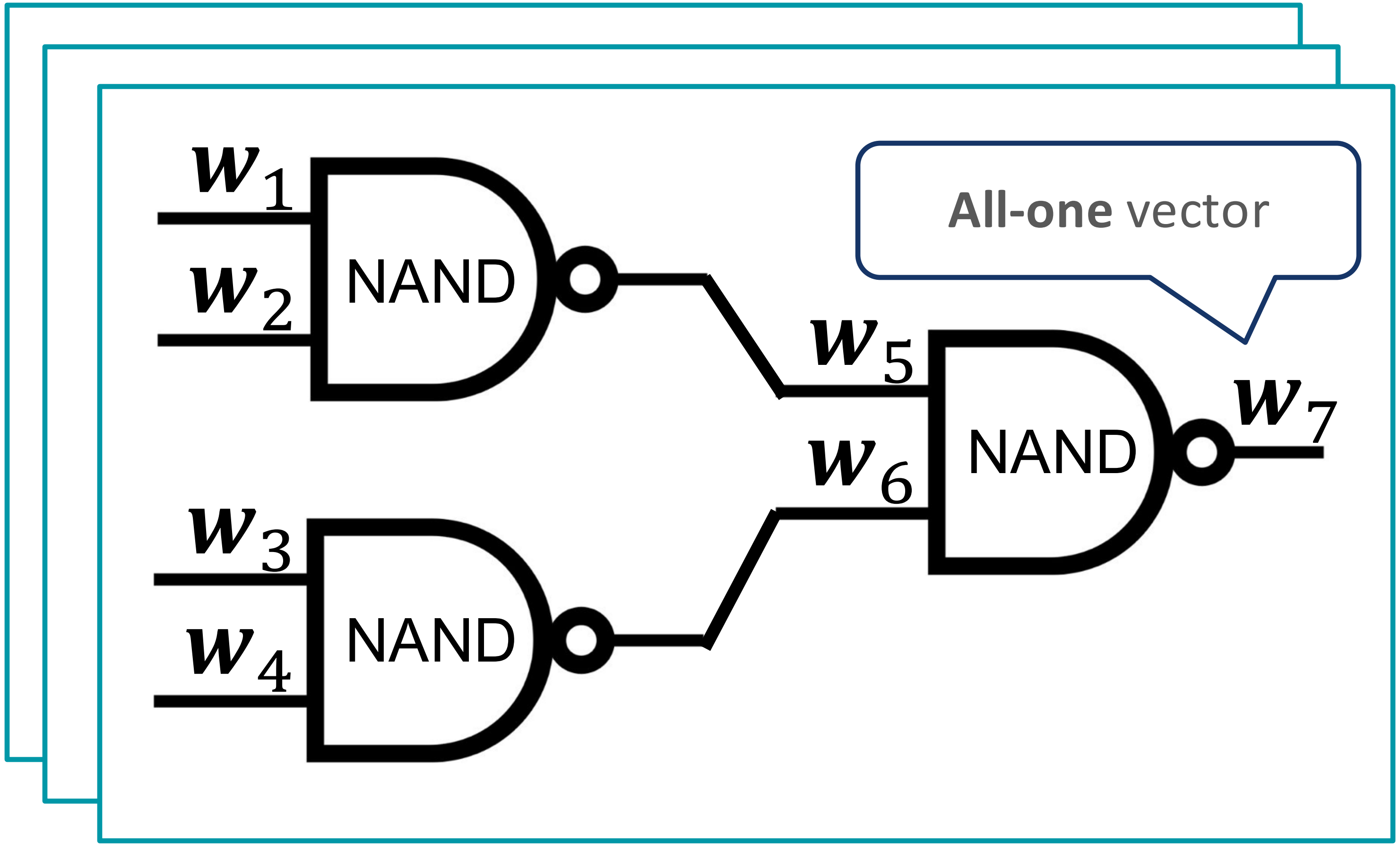

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Validity proofs

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BARG proof: $\{\sigma_i\}$ + validity proofs

Q: How to compute validity proofs?

Let's focus on wire validity proofs

Quadratic Eq-Check over Exponent [WW'22]

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$\mathbb{G}_p$: Subgroup of order p w/ generator g_p

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Caveat: CRS includes $\{[\alpha_i \alpha_j]\}_{i \neq j}$

ℓ^2-size

Q: Check quadratic equations without cross-terms?

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Idea: Vector commitment

Polynomial commitment

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- Linear CRS size
- $O(\ell \log \ell)$ $\mathbb{Z}_N$-ops + $O(\ell)$ $\mathbb{G}$-ops

Interpolation set = Roots-of-unity

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Q: How about other validity proofs?

Similar approach, as relations are quadratic

Comparison with [KZG'10]

[KZG'10]:

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Our result:

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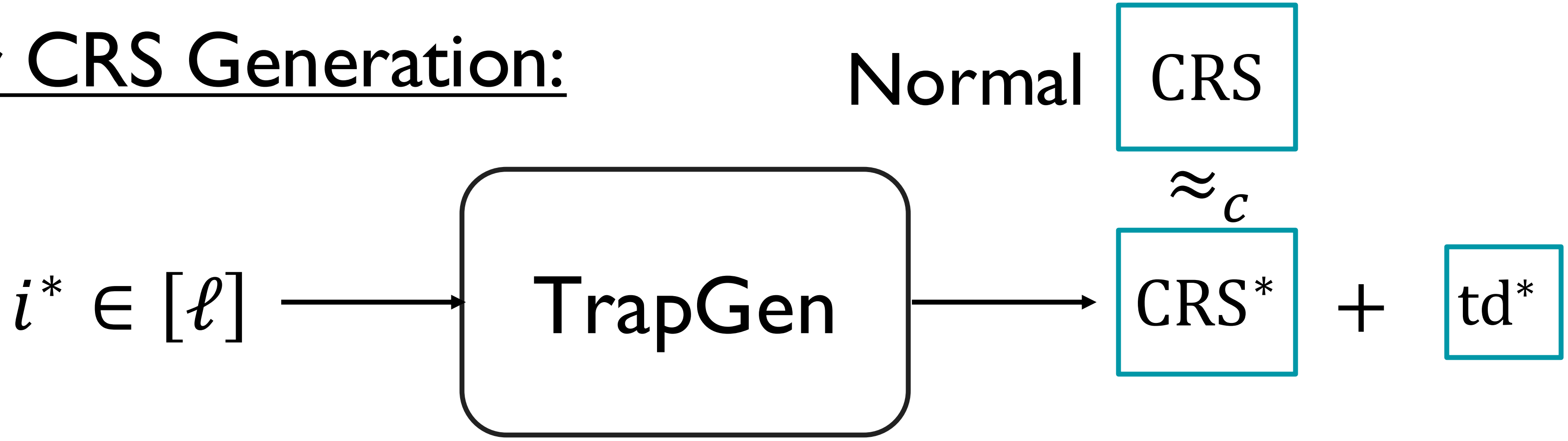

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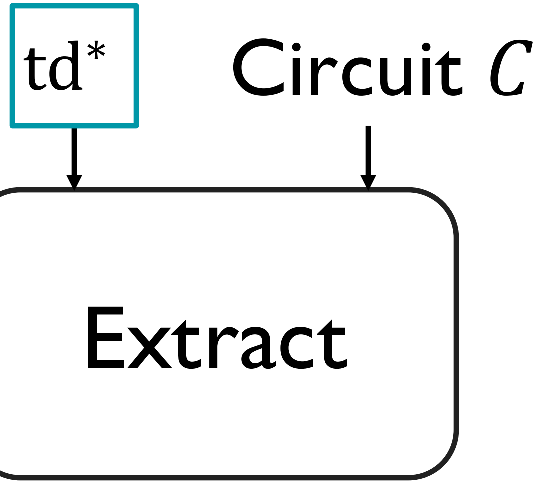

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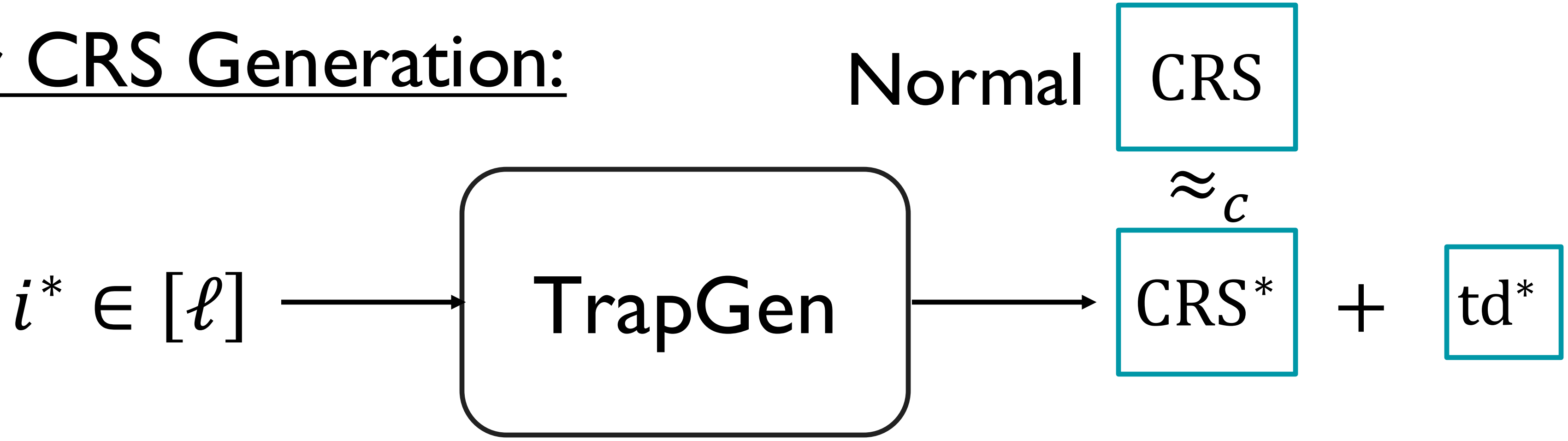

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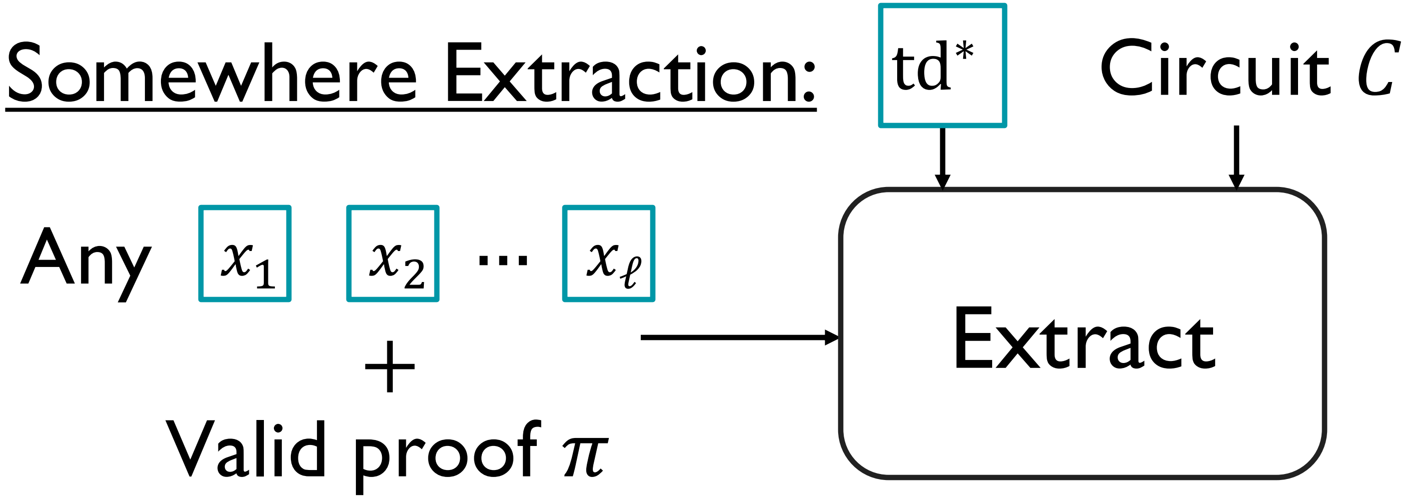

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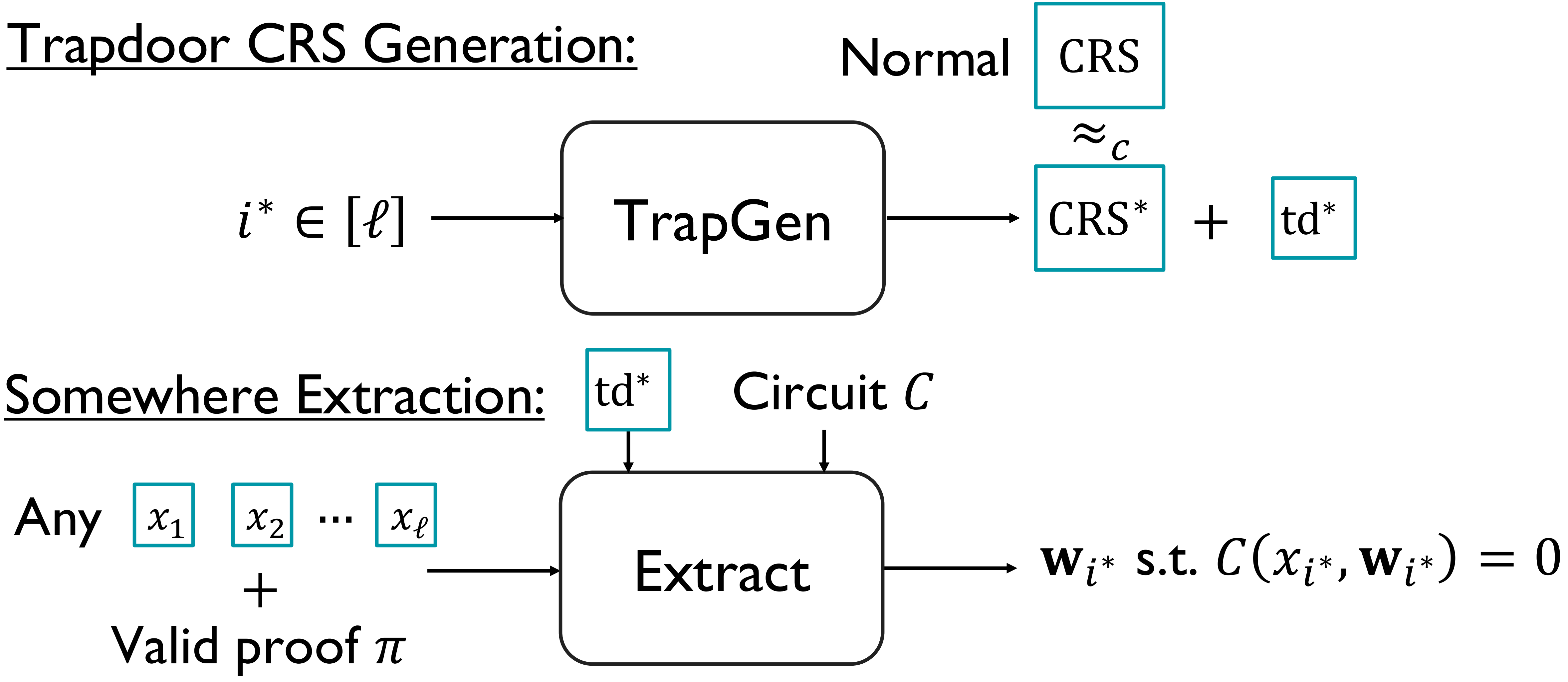

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Subgroup Decision Exponent Assumption:
$[\alpha], \dots, [\alpha^\ell], g_p$
$\approx_c [\alpha], \dots, [\alpha^\ell], g_p g_q$

$\boxed{\mathrm{td}^*} = g_q \in \mathbb{G}_q$

CRS Indistinguishability

Trapdoor CRS Generation: $i^* \in [\ell]$:

$g_p^{\alpha^\ell} g_q^{i*^\ell}$

True in GGM

$\boxed{\mathrm{CRS}^*} = [1]\cdot[1], [\alpha]\cdot[i^*], \ldots, [\alpha^\ell]\cdot[i^{*\ell}]$

$\approx_c$

$\boxed{\mathrm{CRS}} = [1], [\alpha], \ldots, [\alpha^\ell]$

Subgroup Decision Exponent Assumption:

$[\alpha], \ldots, [\alpha^\ell], g_p$

$\approx_c [\alpha], \ldots, [\alpha^\ell], g_p g_q$

$\boxed{\mathrm{td}^*} = g_q \in \mathbb{G}_q$

Somewhere Extraction

$\boxed{\mathrm{CRS}^*} = [1] \cdot [1], [\alpha] \cdot [i^*], \ldots, [\alpha^\ell] \cdot [{i^*}^\ell]$

$\boxed{\mathrm{td}^*} = g_q :$

Somewhere Extraction

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A valid wire commitment from the prover is of the form:

$$\sigma_{\boldsymbol{w}} = [\phi(\alpha)] \cdot [\phi(i^*)] = [\phi(\alpha)] \cdot [w_{i^*}]$$

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Use td^* to project $\sigma_{\boldsymbol{w}}$ onto subgroup $\mathbb{G}_q$

Allow extraction of $w_{i^*} \in \{0, 1\}$

$\boxed{\mathrm{td}^*} = g_q$: $\quad e\left(g_q, [\phi(\alpha)] \cdot [w_{i^*}]\right) = e\left(g_q, g_q\right)^{w_{i^*}}$

Summary

- Extend [WW'22] to the polynomial setting
- Linear-size CRS, quasilinear prover time, black-box crypto
- Security from falsifiable assumptions

Open Problems:

Extend to prime-order groups?

Lattice-based constructions?

THANK YOU

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