LatticeFold

Memory-Efficient Proving at Scale

Binyi Chen

Joint work with Dan Boneh

Stanford University

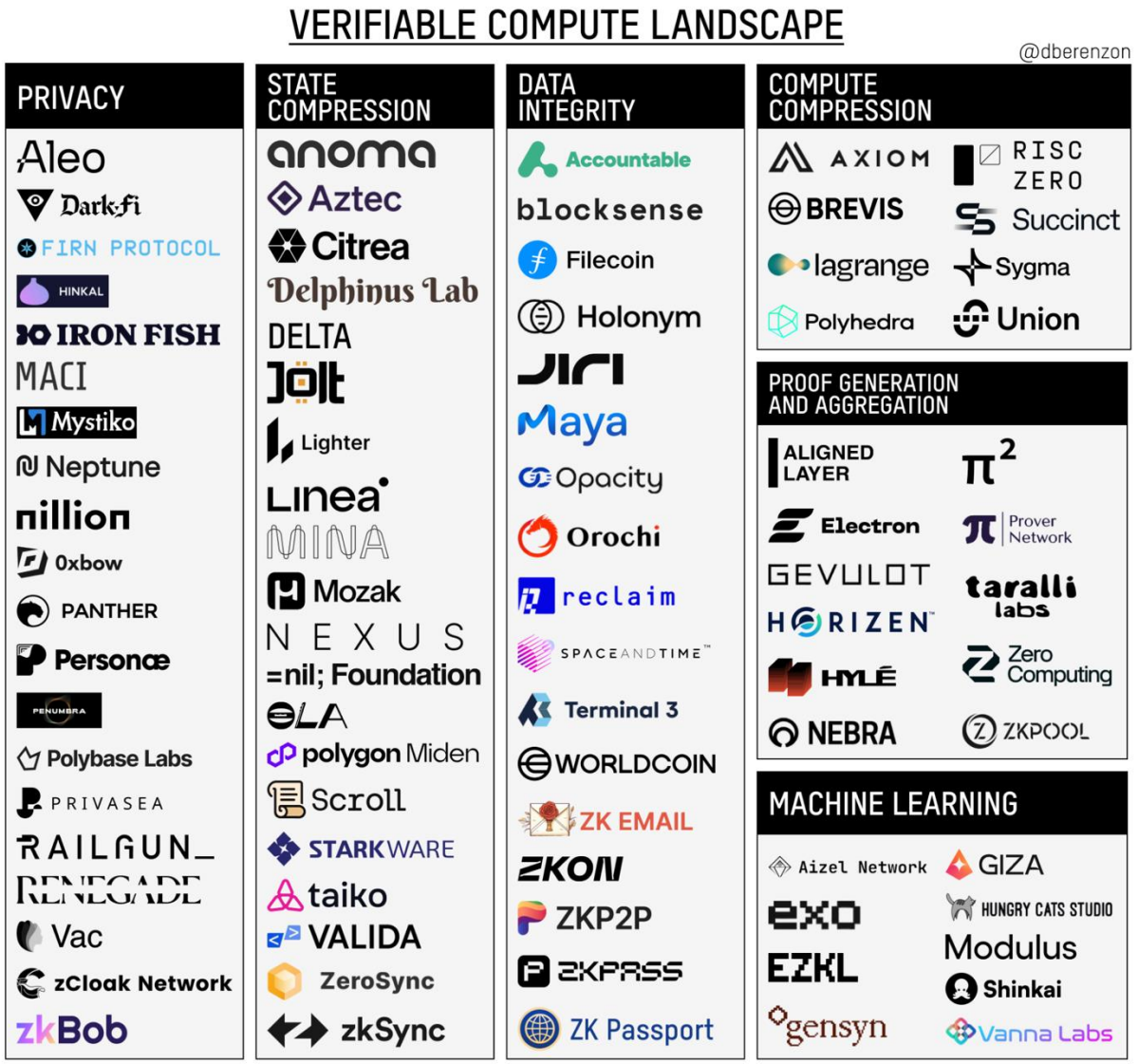
ZK-SNARKs: Advanced Applications

Applications

- zkVM/zkML
- Image/Video Provenance
- ZK Wallet/Passport
- Data Availability
- Decentralized Storage
- .....

Common Theme

Large-scale computation



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Common Theme

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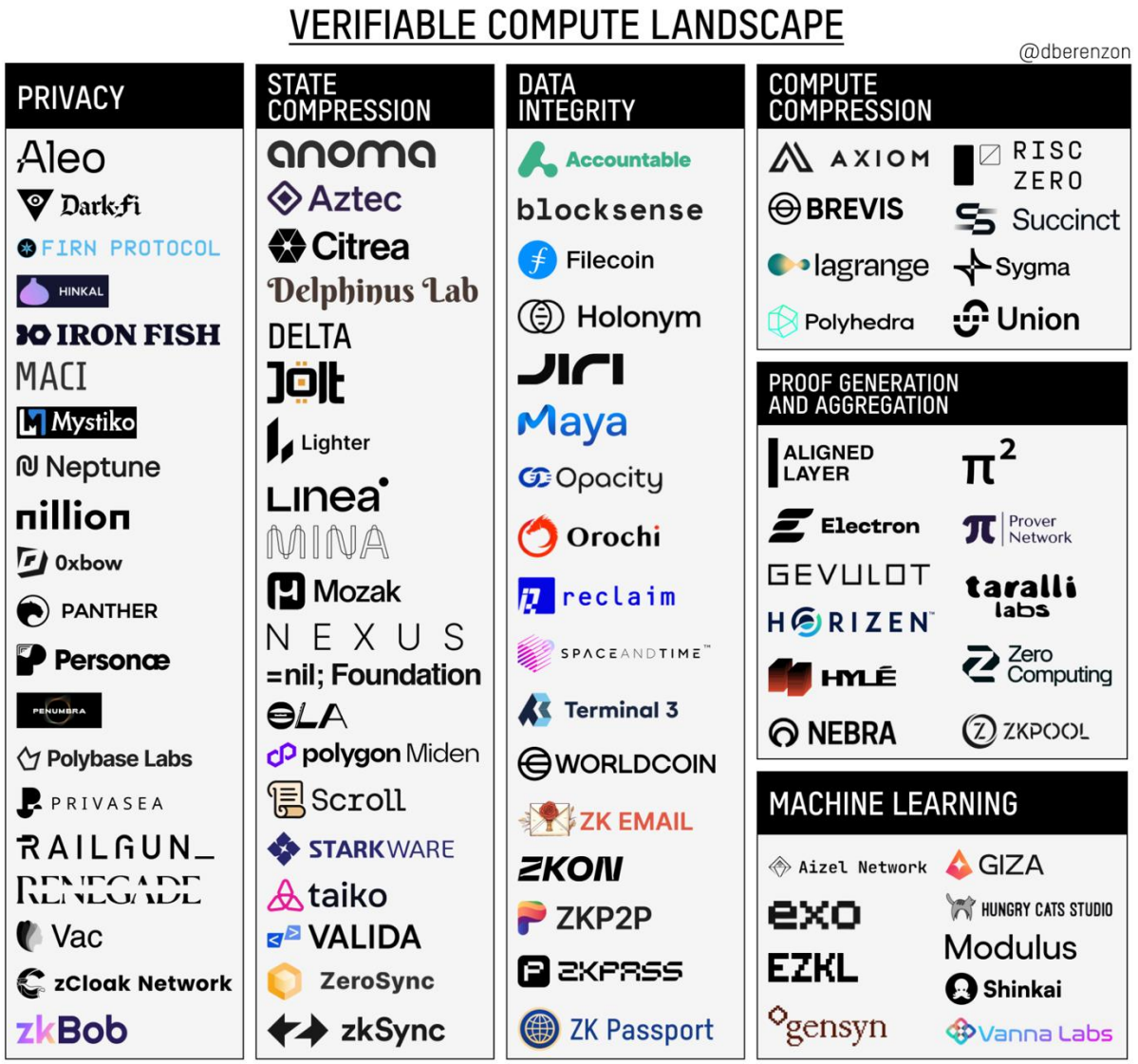
Design Requirements

Scalable prover + post-quantum security

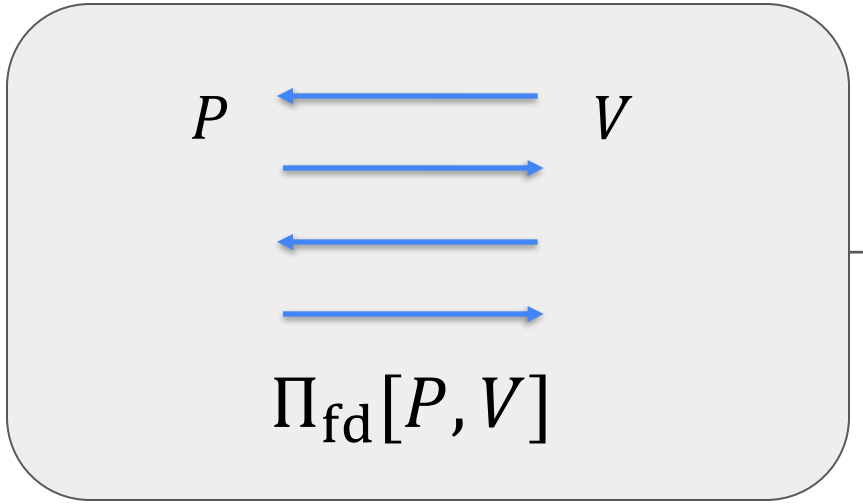


Folding Schemes [BGH'19, BCLMS'20, KST'21]

$|R_{NP}| \approx |R_{acc}|$

$(x_1, w_1) \in ? R_{NP}$

$(x_2, w_2) \in ? R_{acc}$



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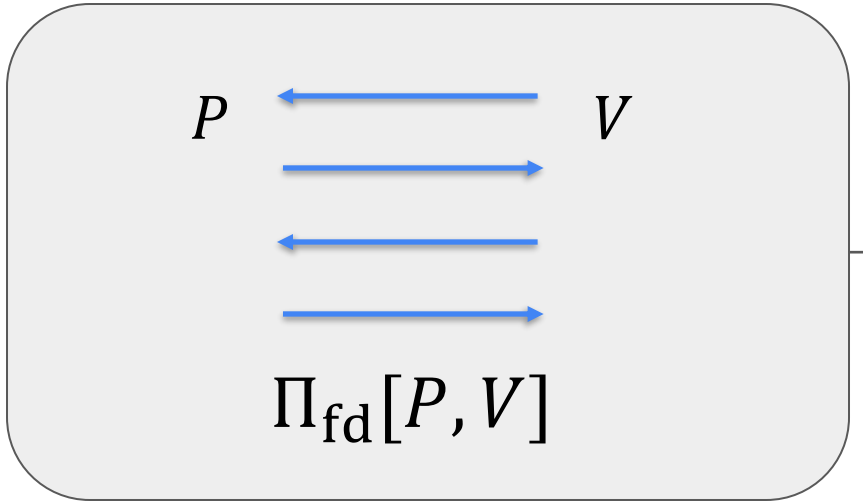

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Non-Interactive Version:
$FS^{H}(\Pi_{fd}[P, V])$

Folding Schemes [BGH'19, BCLMS'20, KST'21]

Complete + Knowledge sound

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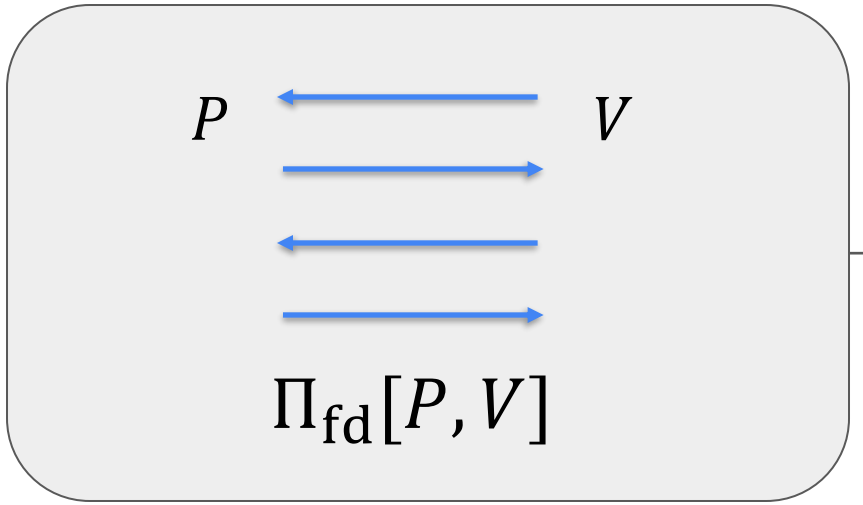

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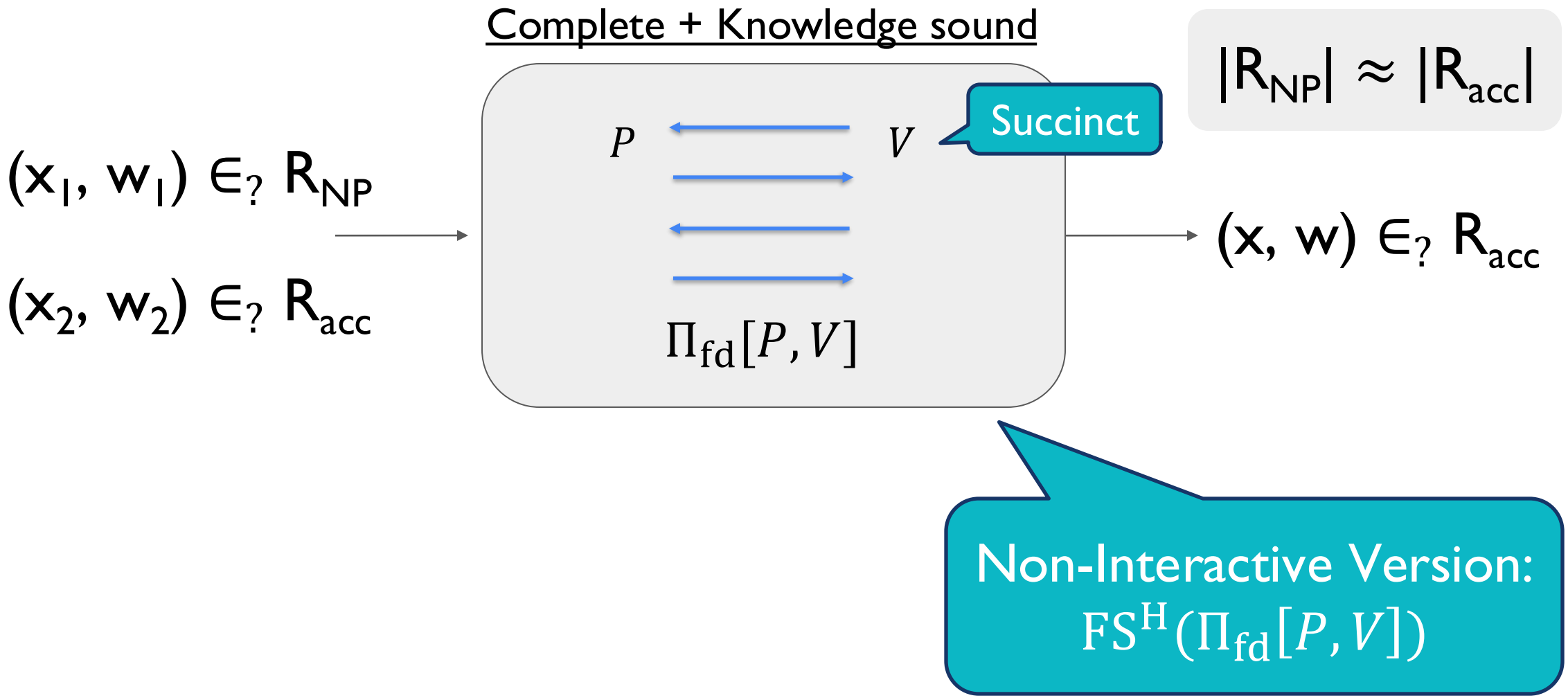

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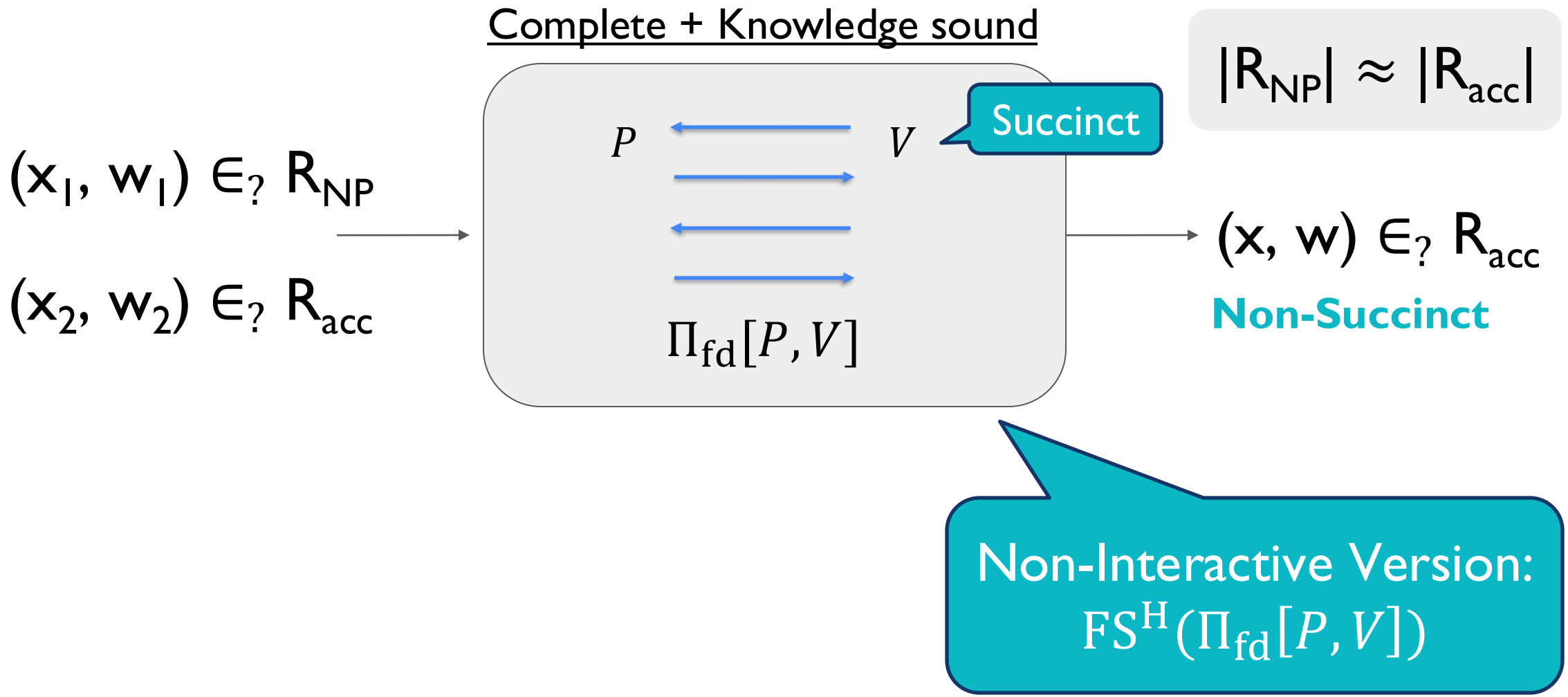

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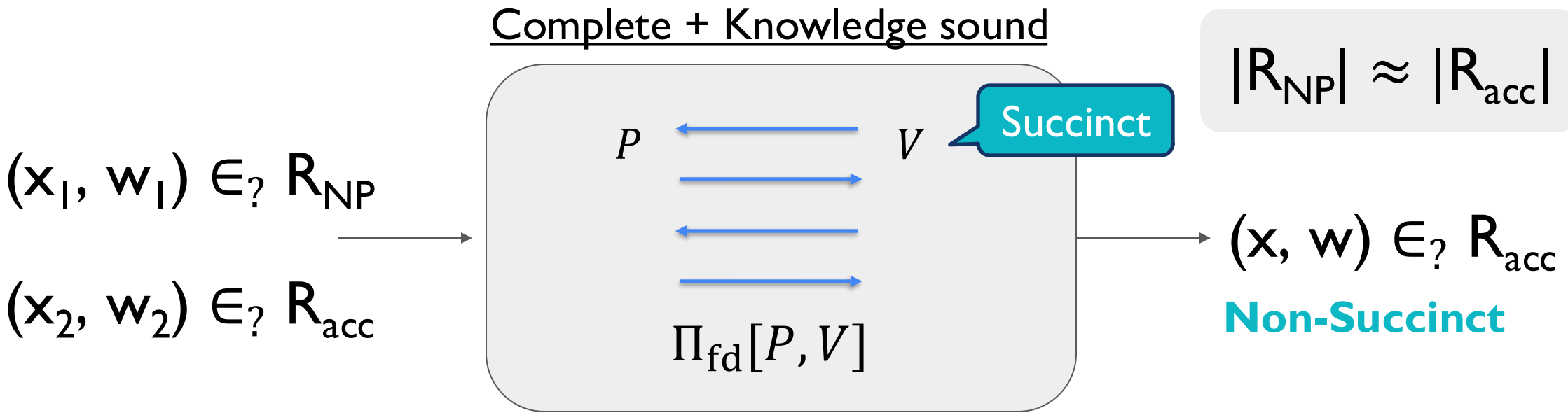

Advantages:

- Much faster than SNARKs
- Boost SNARK efficiency

Non-Interactive Version:
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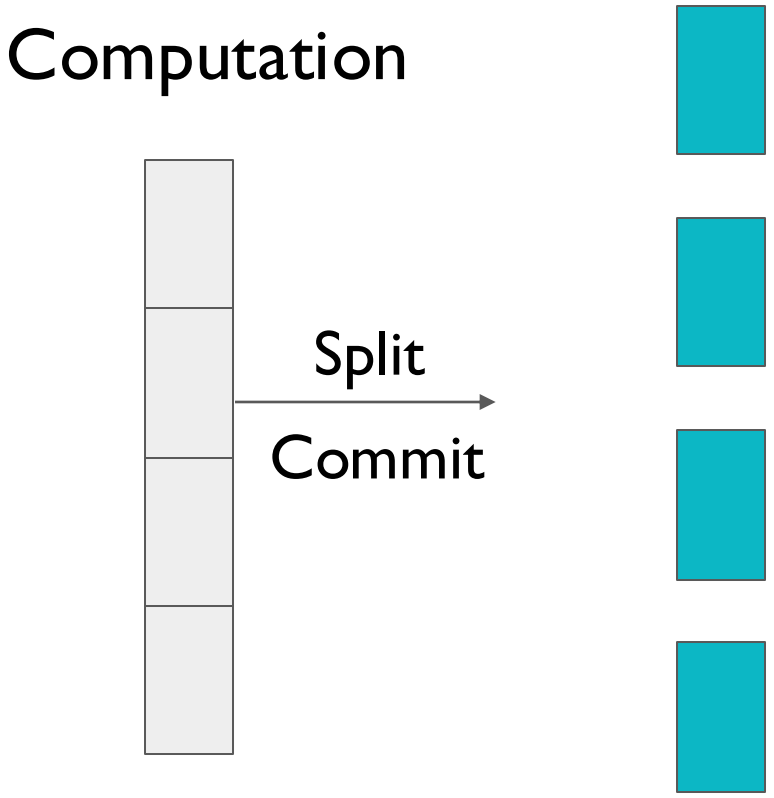
Folding-based SNARKs [BGH'19, COS'20, NDCTB'24…]



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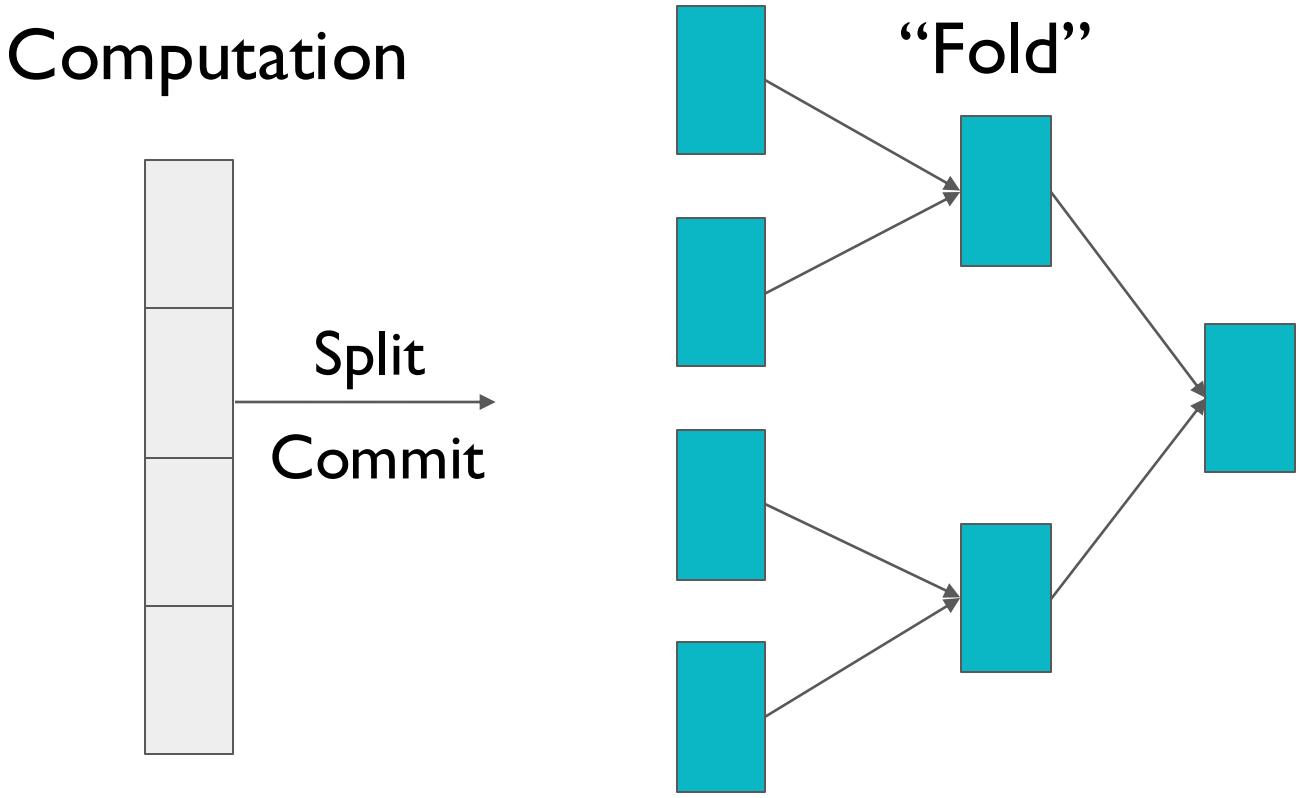

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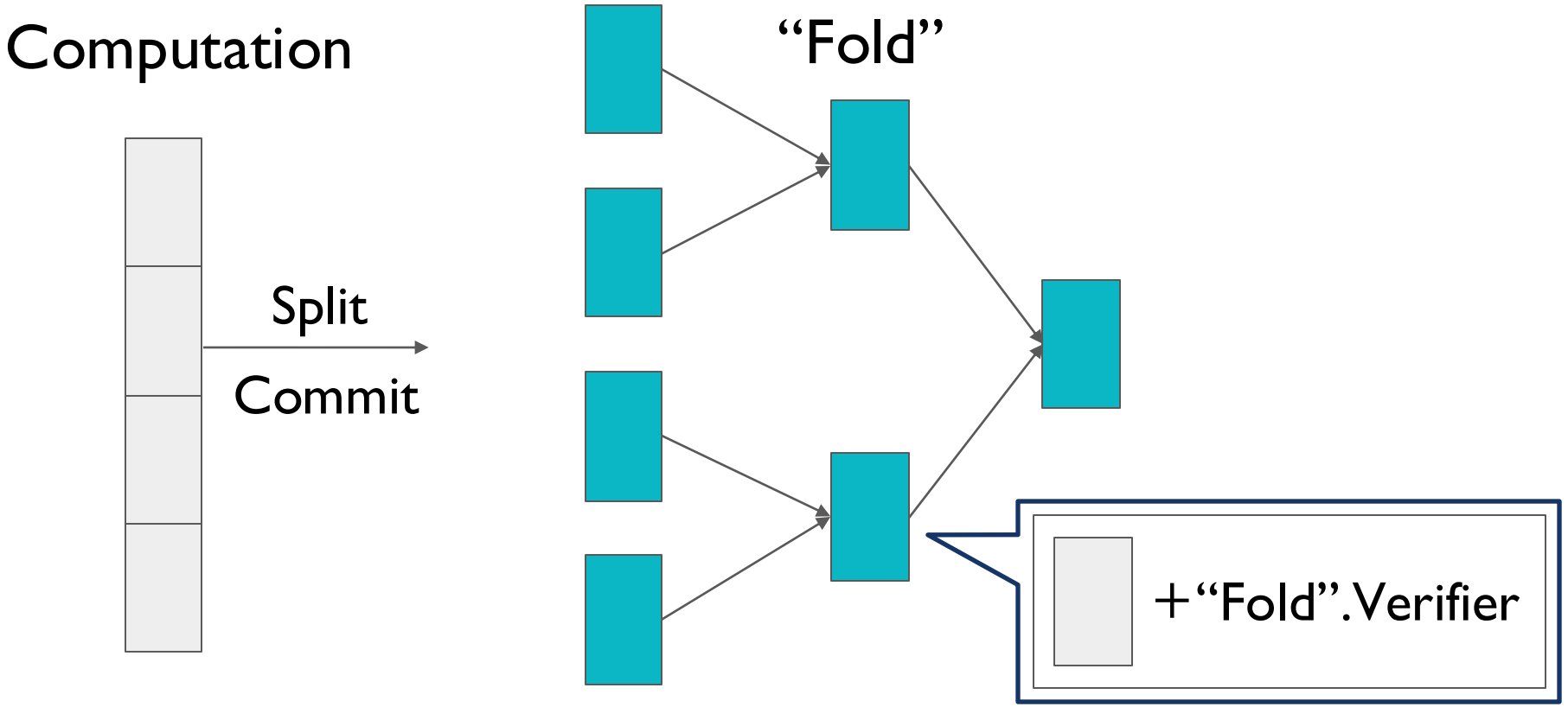

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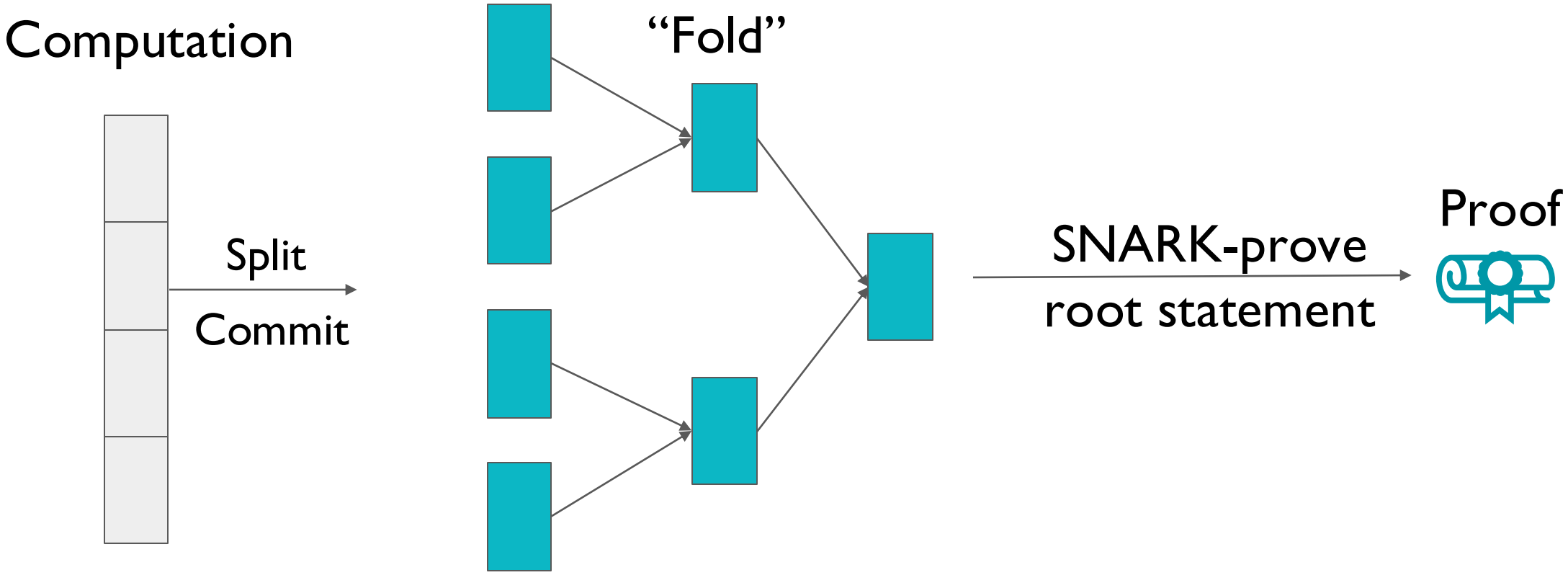

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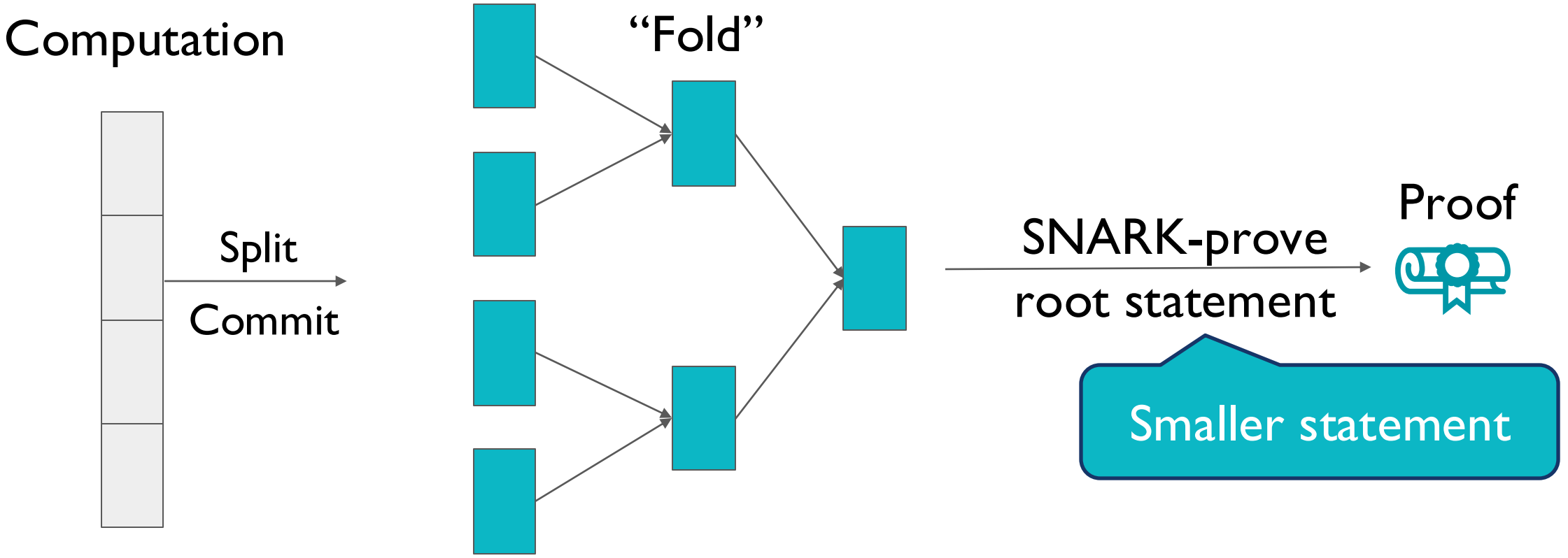

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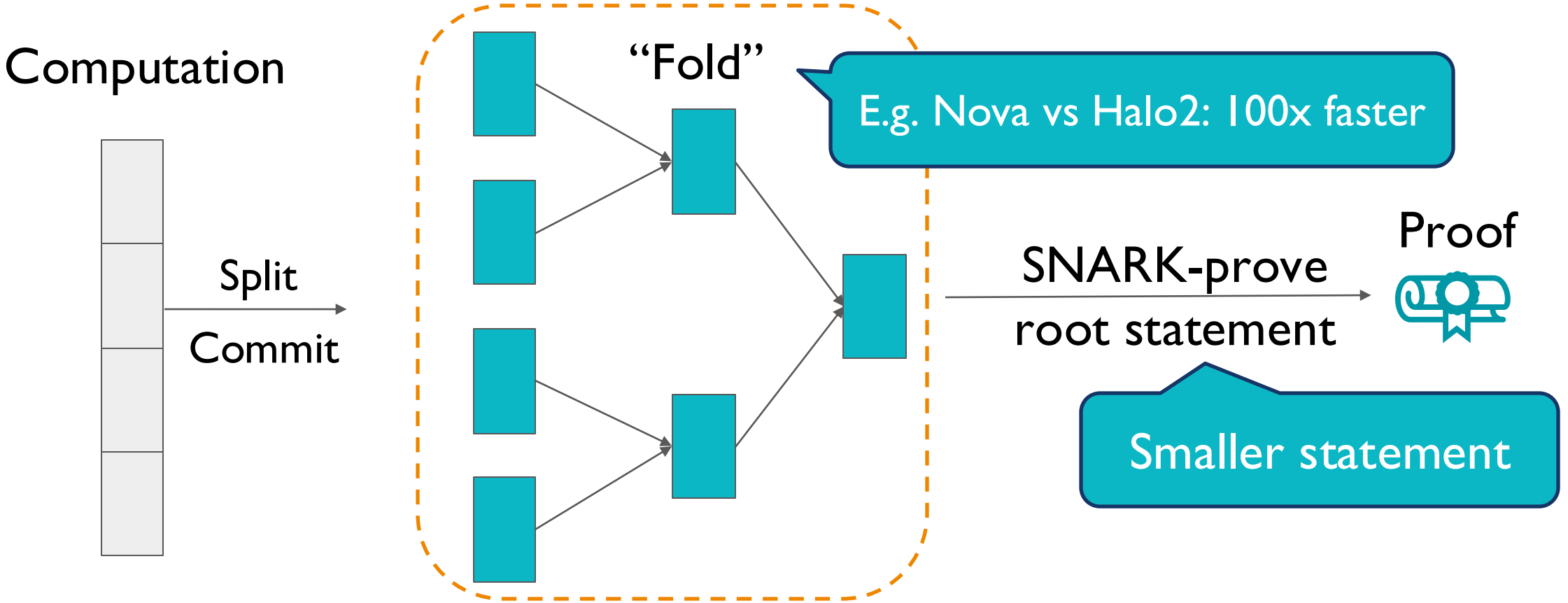

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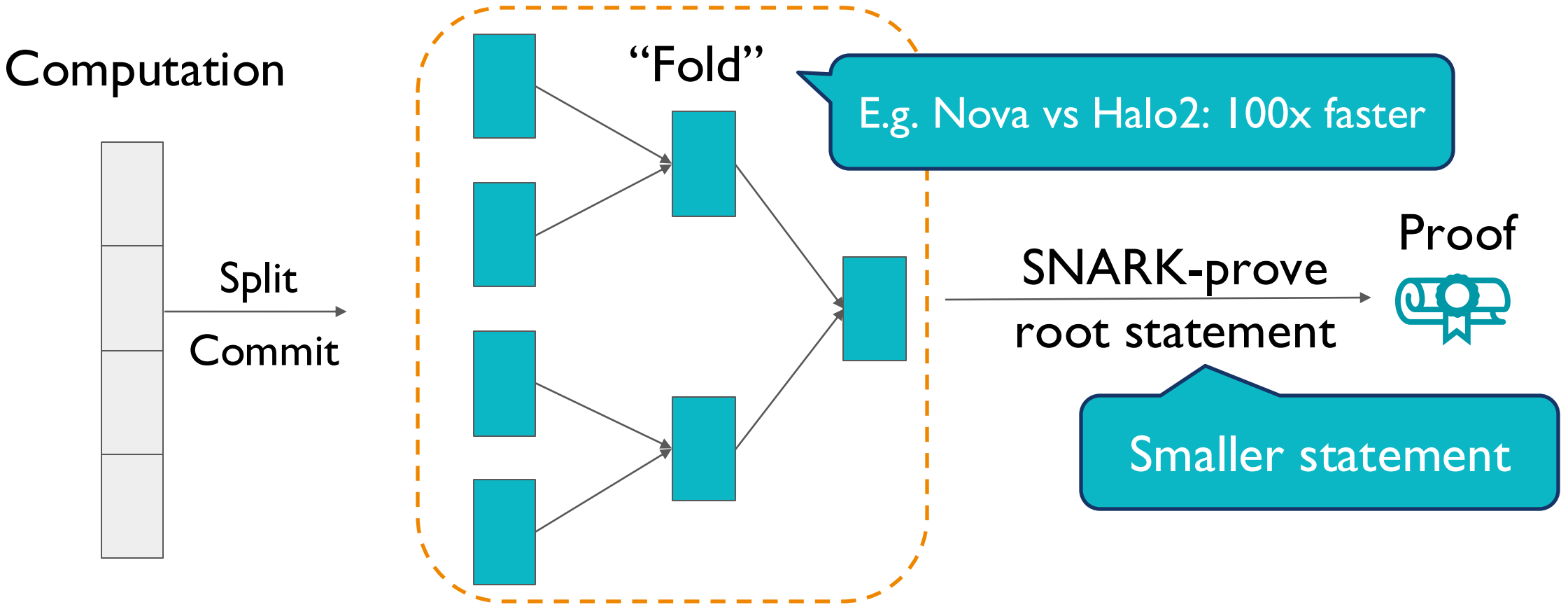
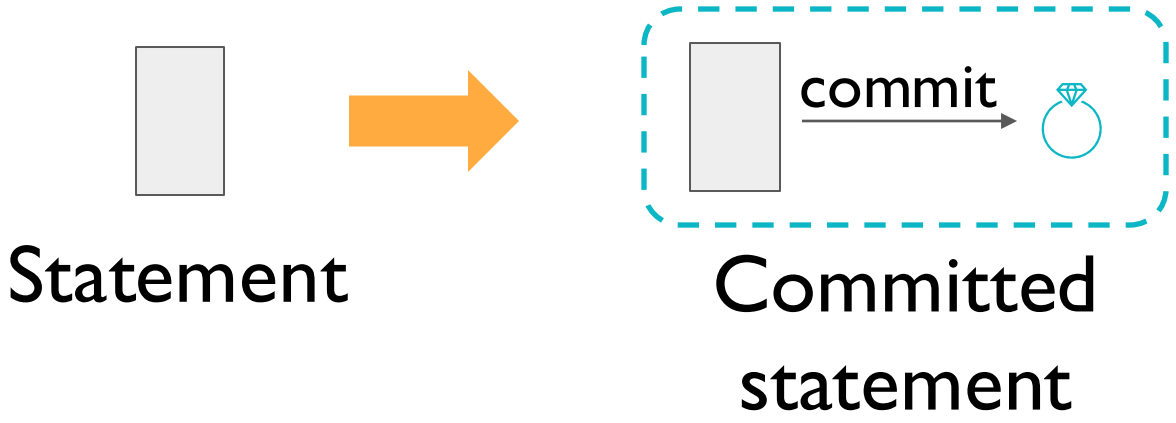


Qs: Which commitment should we use?

The Choice of Commitments

Statement

The Choice of Commitments



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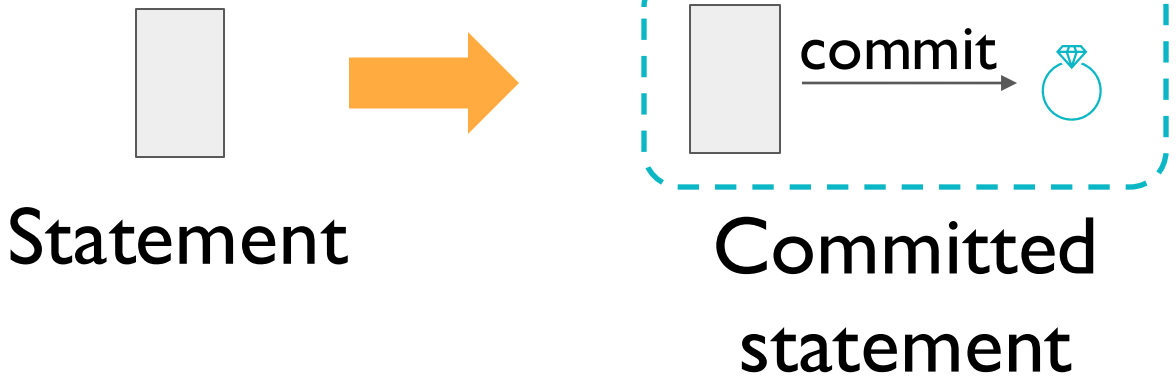
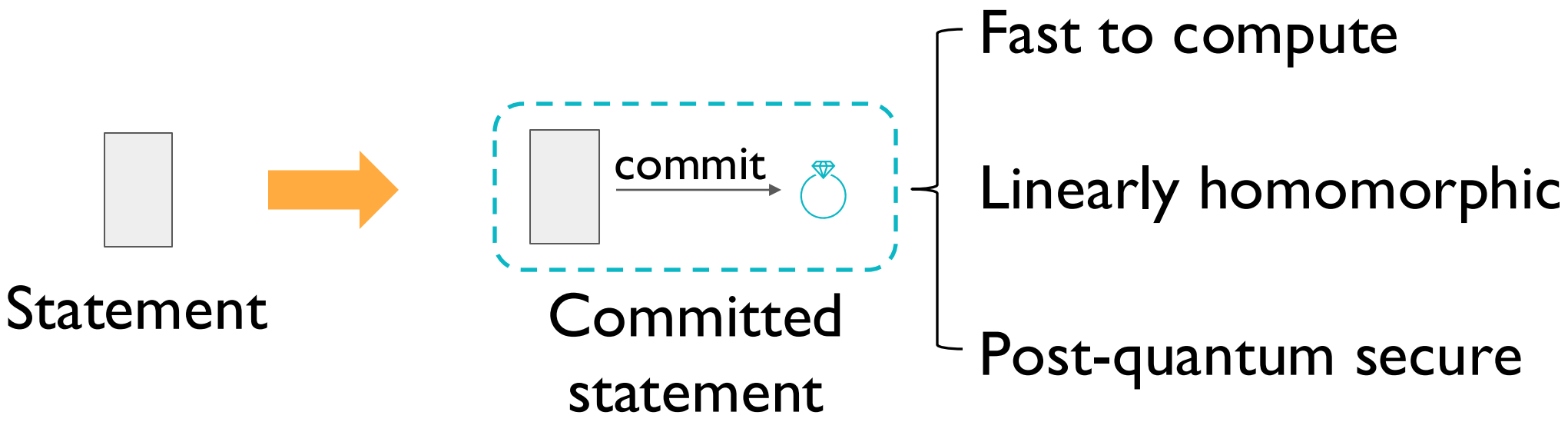


- Fast to compute
- Linearly homomorphic
- Post-quantum secure

The Choice of Commitments

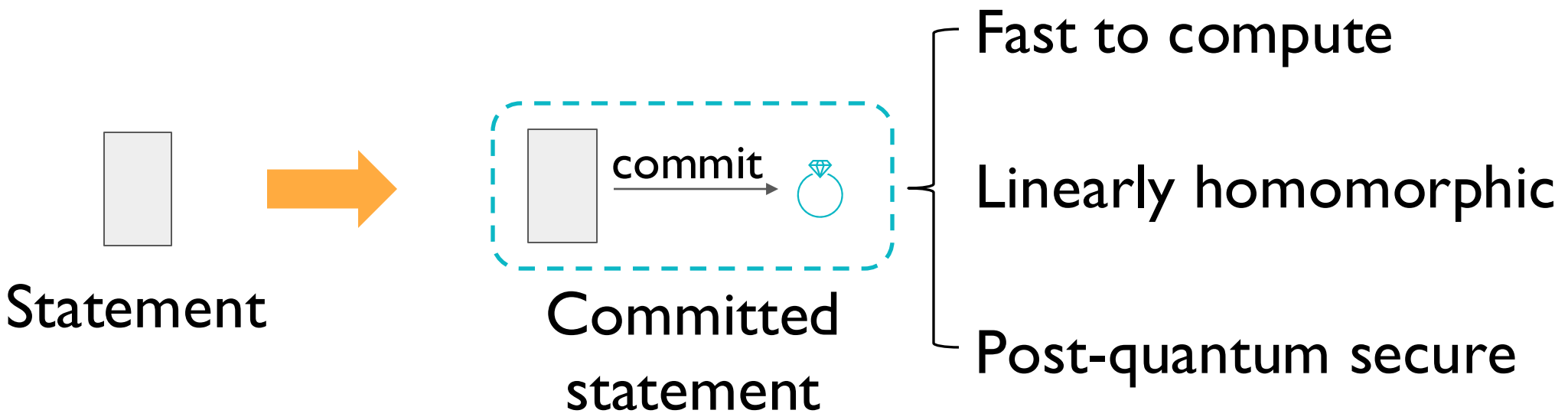


Before: Pedersen commitments [KST'21, BCLMS'21, BC'23, KS'23…]

Pros:

- Linearly homomorphic

The Choice of Commitments



Before: Pedersen commitments [KST'21, BCLMS'21, BC'23, KS'23…]

Pros:

- Linearly homomorphic

Cons:

- Pre-quantum
- Require expensive cycle-curves

Q: A folding scheme from a faster and PQ-secure commitment?

Our Contribution

Main Result

- The first plausibly PQ-secure folding scheme from Ajtai hashes
- Commitment space ≈ Witness space ⇒ Native verifier circuit

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Follow-up Works:

- LF+ [BC'25], Symphony [C'25]
- Neo [NS'25]
- RoK-and-Roll [KLNO'25]
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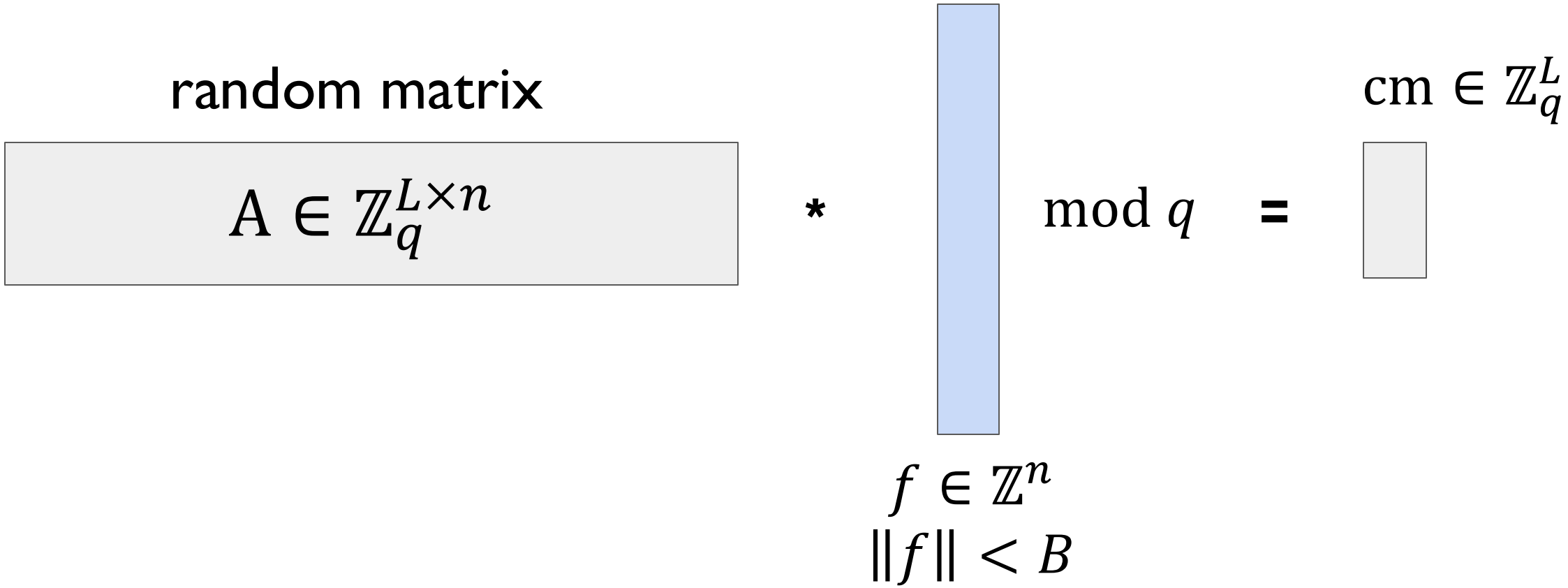
Hash-based Schemes:

[BMNW'25a, BMNW'25b, BCFW25, BMMS'25…]

- Based on Merkle-commitments
- PQ-secure and fast committing
- More expensive verifier circuit

Lattice-based Commitments [Ajtai'96]



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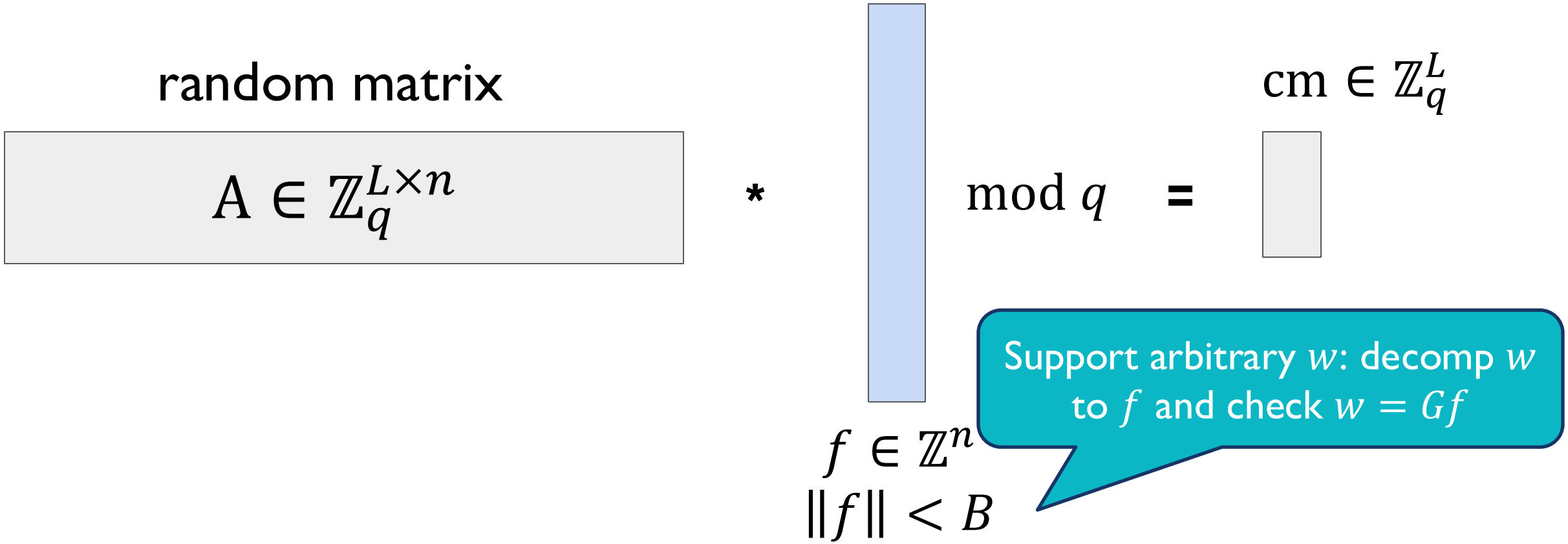

Lattice-based Commitments [Ajtai'96]

"Semi"-linearly homomorphic

random matrix

$$A \in \mathbb{Z}_q^{L \times n} \quad * \quad \begin{matrix} f \in \mathbb{Z}^n \\ \|f\| < B \end{matrix} \quad \bmod q \quad = \quad \mathrm{cm} \in \mathbb{Z}_q^L$$

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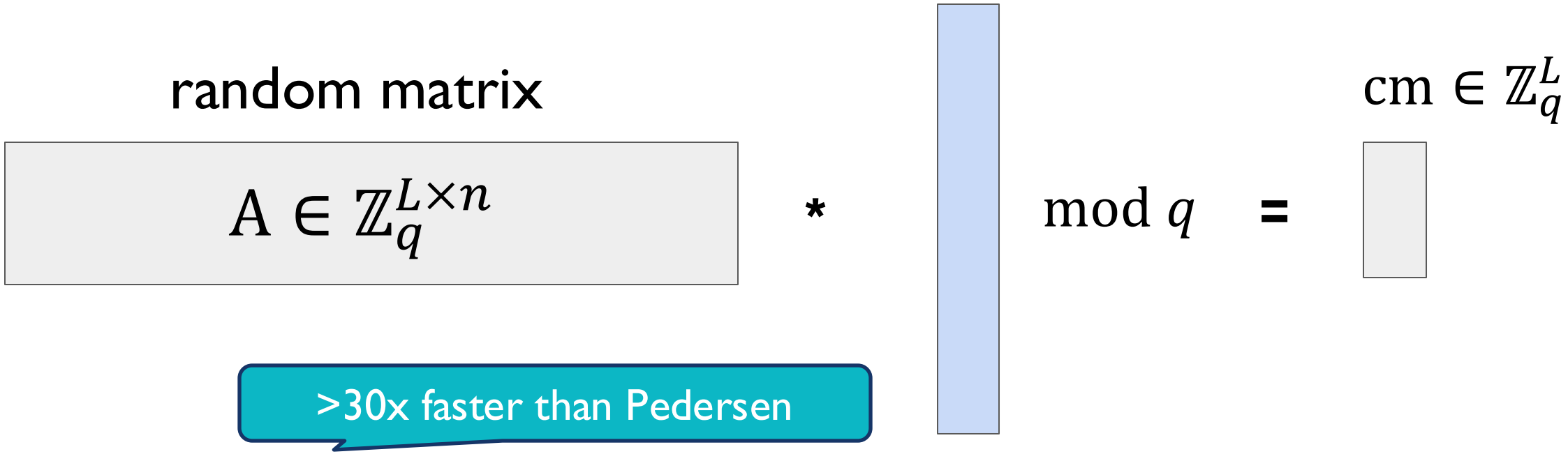


>30x faster than Pedersen

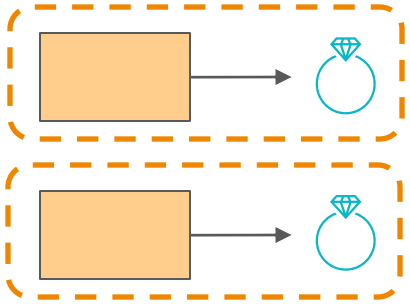
SWIFFT [LMPR'08]

A faster variant over R_q!

Q: Plug to existing folding templates?

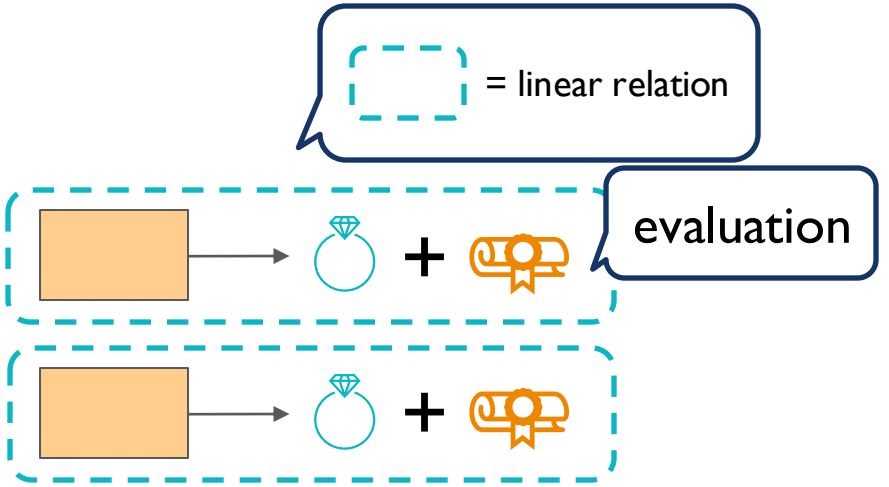
A Typical Folding Framework [KS'23]

Step 1: Commit



Witnesses + Comm

Step 2: Linearization



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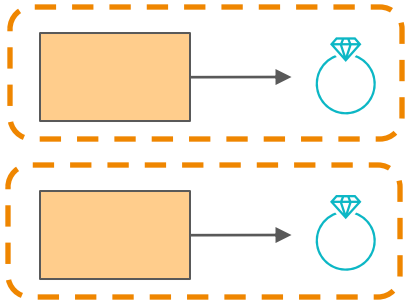

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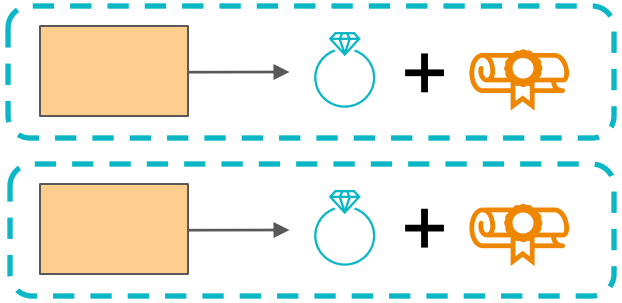
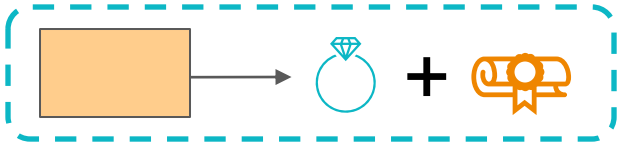


Step 3: Fold



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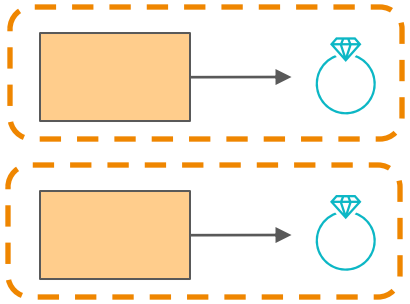

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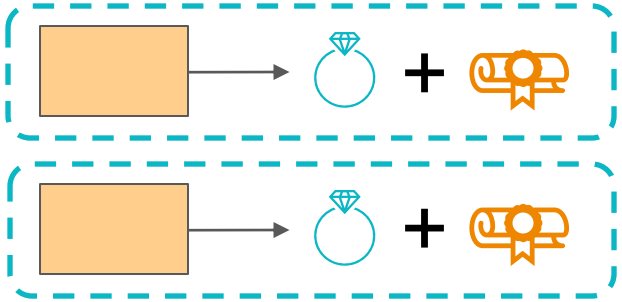

rand. lincomb

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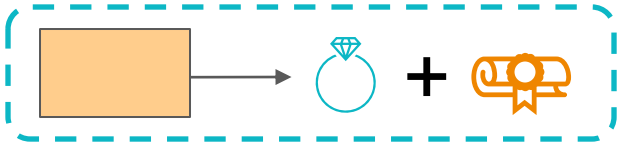

Challenges in using lattice commitments

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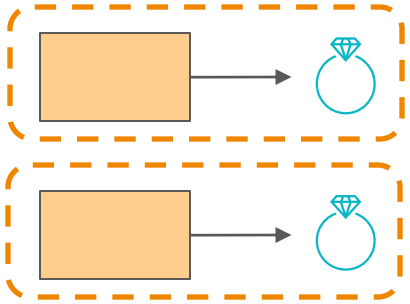

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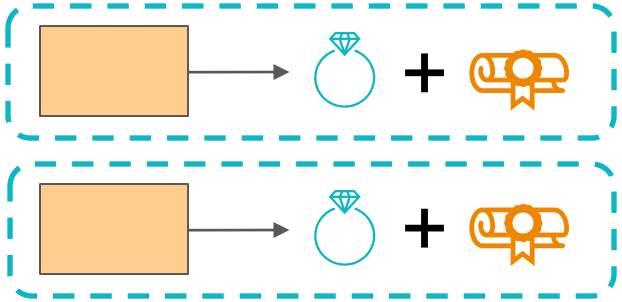
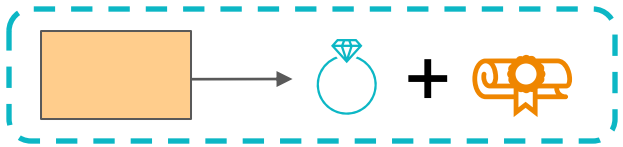


rand. lincomb

Step 3: Fold

Challenge 1: Control norm blowup



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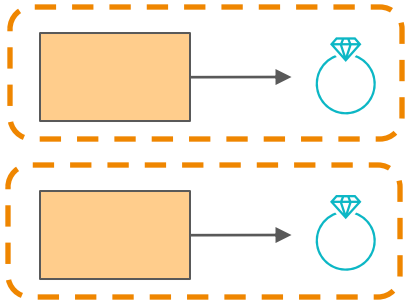
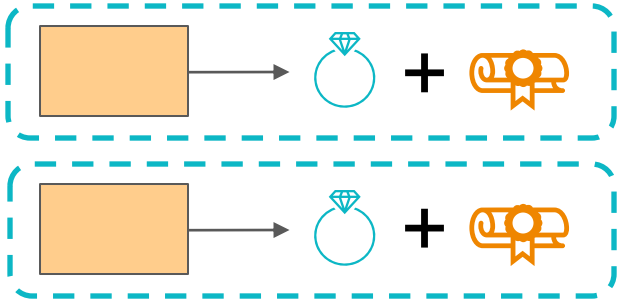


Witnesses + Comm

Step 2: Linearization

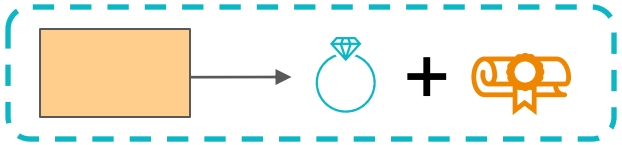


Challenge 2: Extract low-normed witnesses

rand. lincomb

Step 3: Fold

Challenge 1: Control norm blowup



Challenges in using lattice commitments

A New Lattice-Folding Framework

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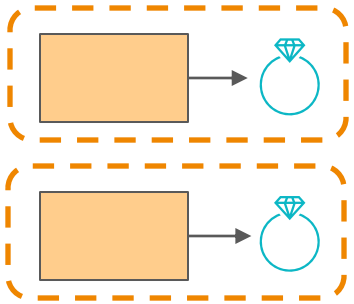
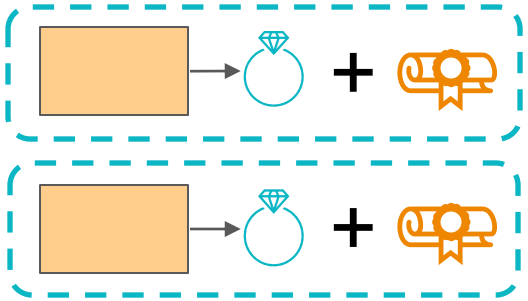


Step 2: Linearize



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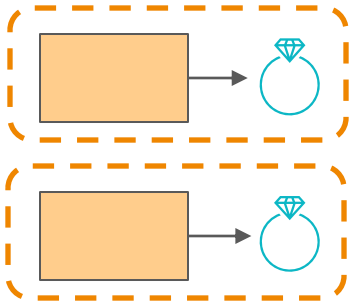

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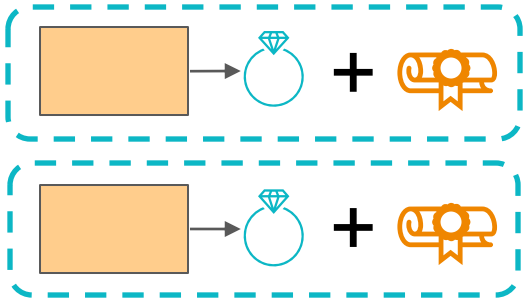
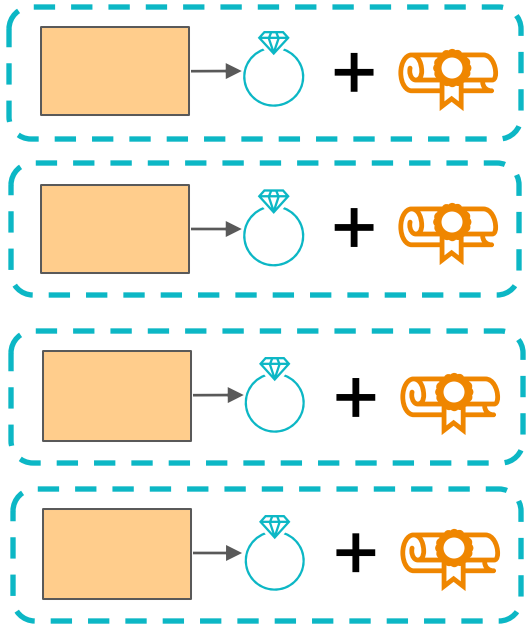
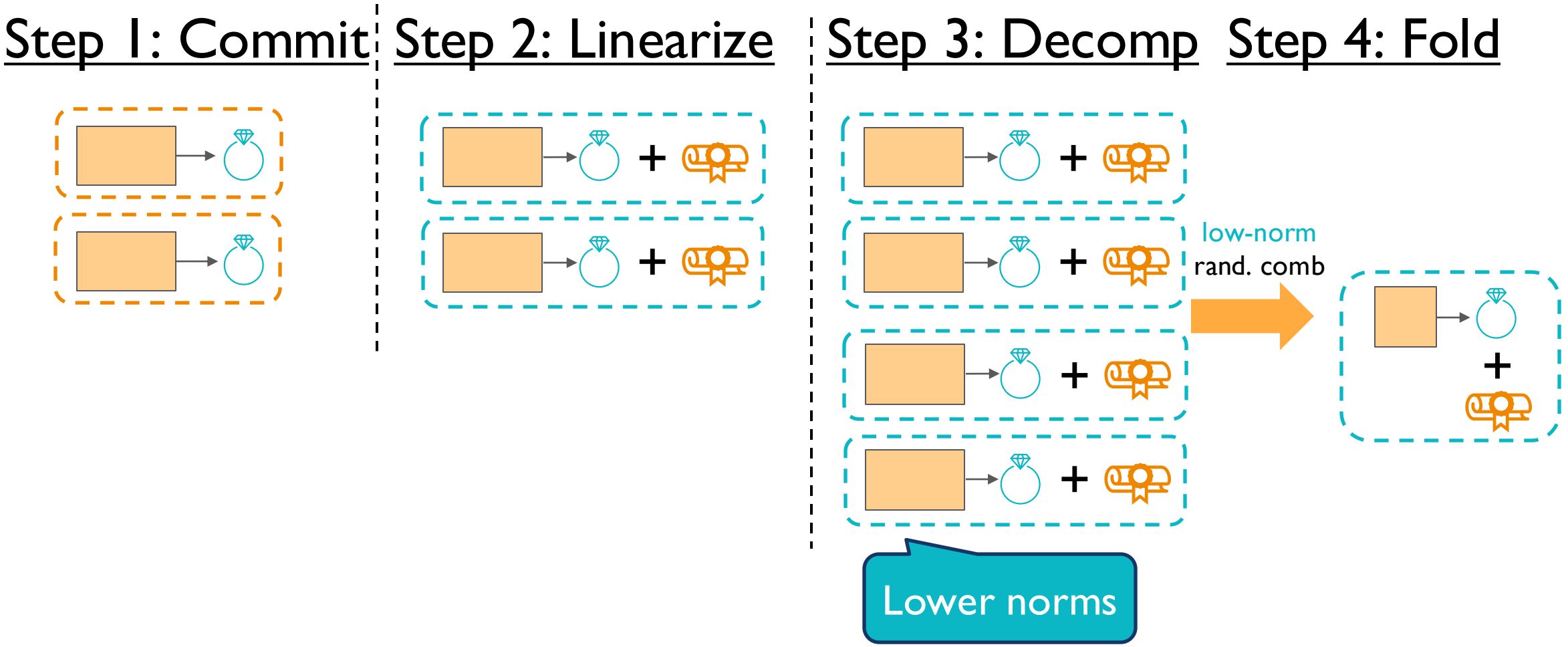


Step 3: Decomp



Lower norms

A New Lattice-Folding Framework



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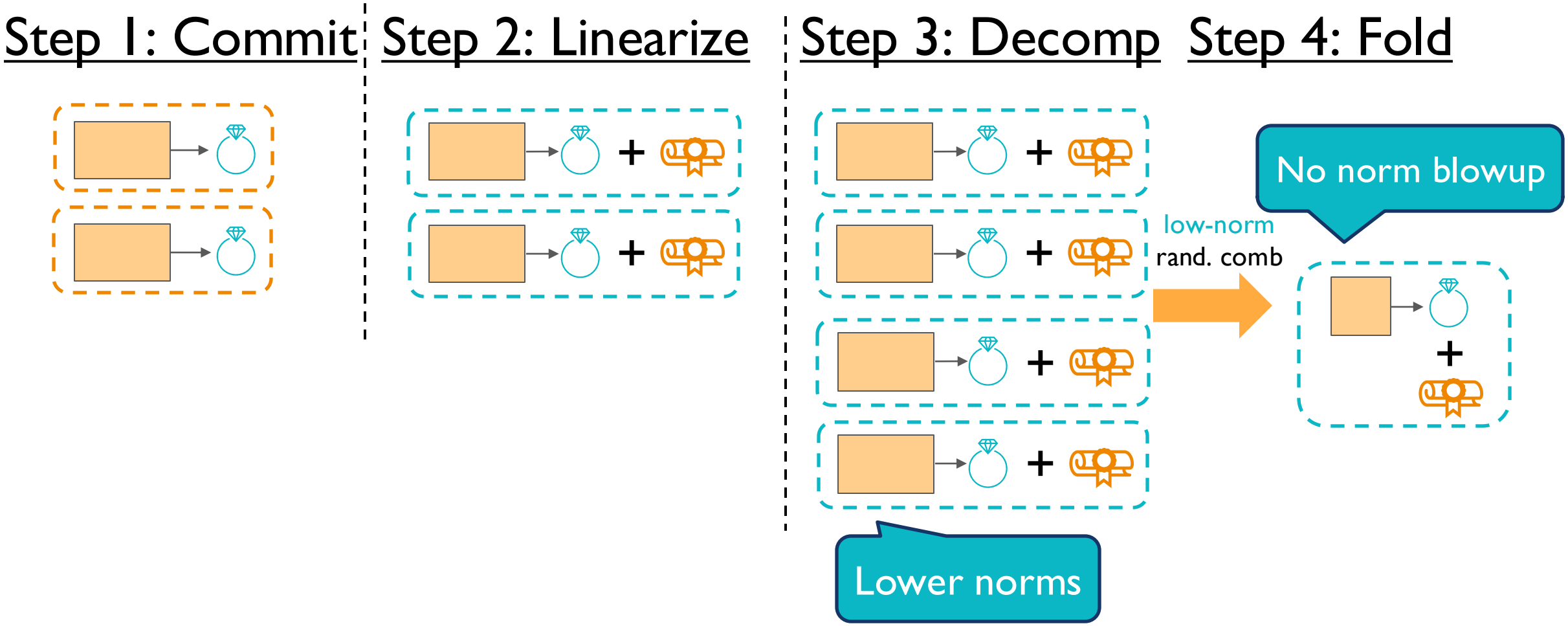
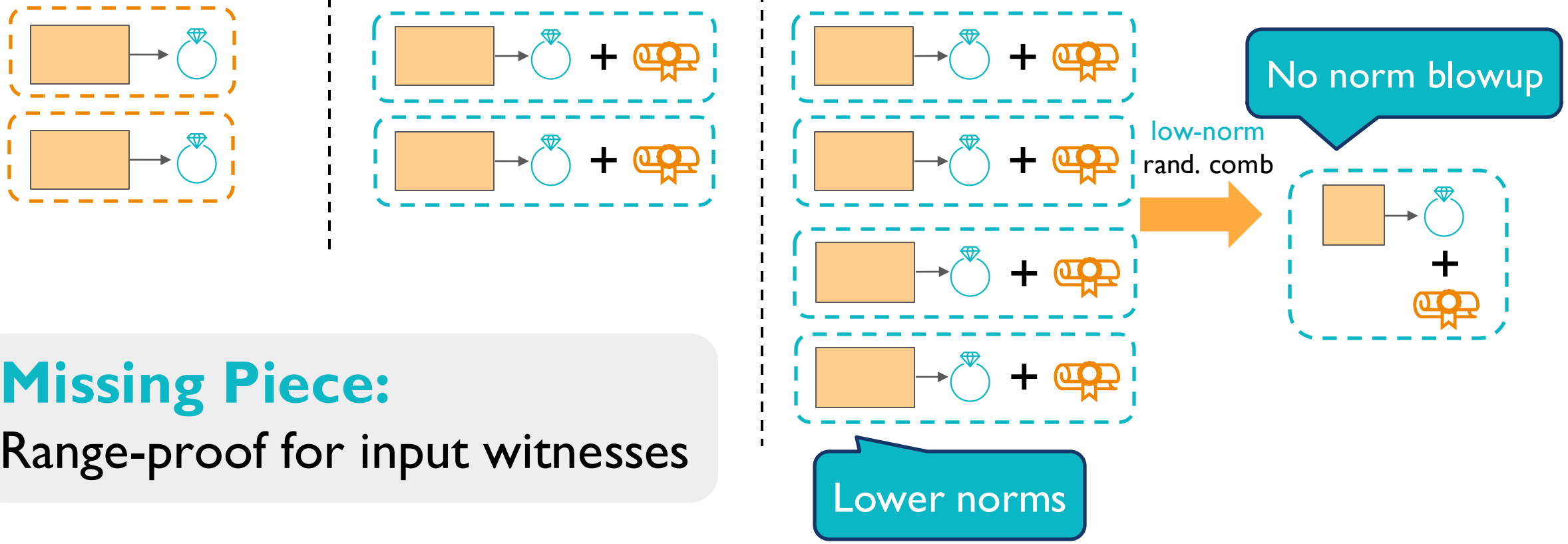


A New Lattice-Folding Framework

Step 1: Commit | Step 2: Linearize | Step 3: Decomp | Step 4: Fold



Missing Piece:

Range-proof for input witnesses

A New Lattice-Folding Framework

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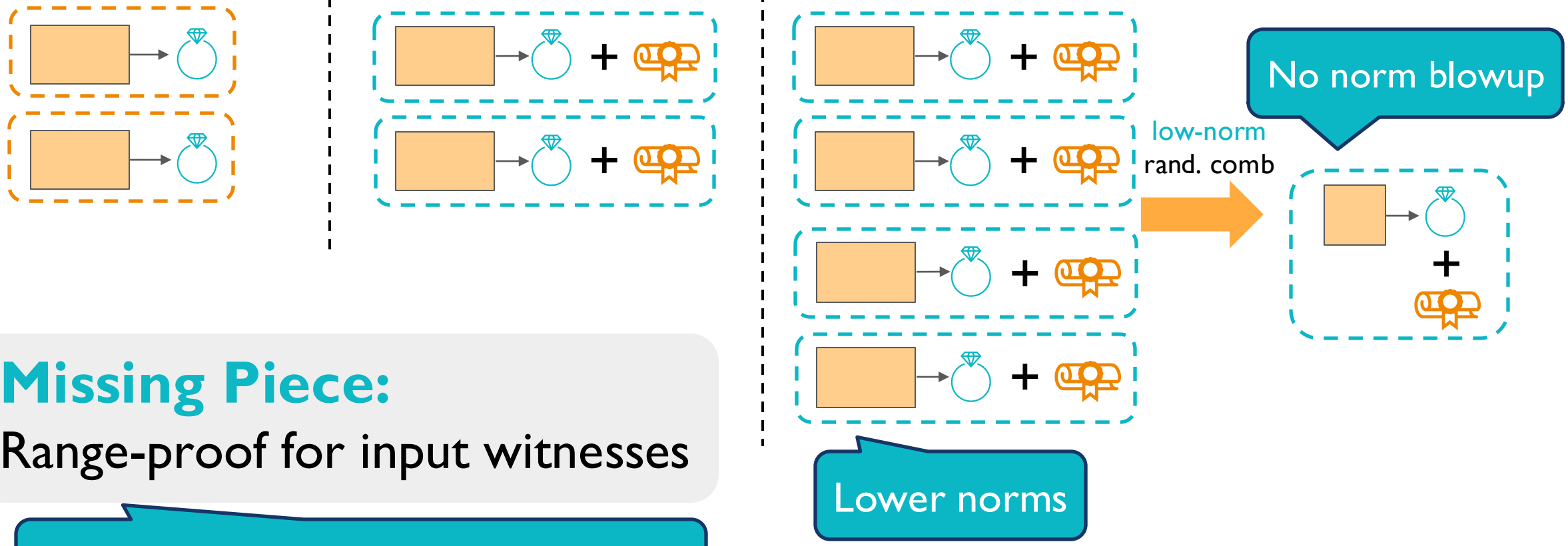


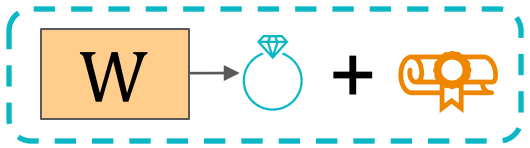
Missing Piece:

Range-proof for input witnesses

Challenge: Sublinear verification

Rephrasing Range Statements

Range statement:



where each entry of W is in $[0, b)$

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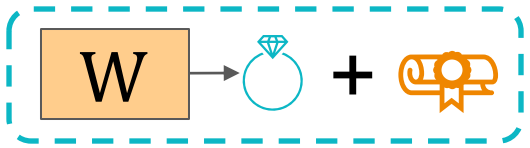


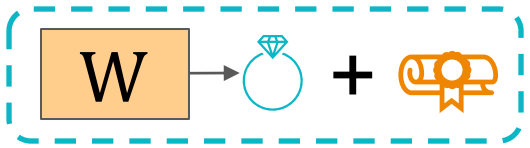
b is small

where each entry of W is in $[0, b)$

Rephrasing Range Statements

Range statement:

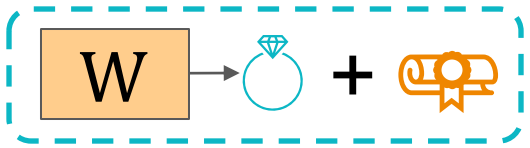
Generalize to $(-b, b)$



where each entry of W is in $[0, b)$

Rephrasing Range Statements

Range statement:



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Observation:

$$\mathrm{W}_{i,j} \in [0, b) \cap \mathbb{Z} \iff F(i,j) := \prod_{t=[0,b)} \left(\mathrm{W}_{i,j} - t\right) = 0 \bmod q$$

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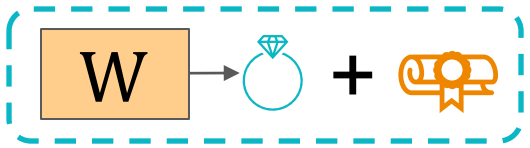

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Degree-b equation

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Range Proof from Sumcheck

Observation:

$$\forall (i,j) \in \mathcal{D}: \quad F(i,j) = 0$$

[Set'20, CBBZ'23]

Sumcheck [LFKN'92]

Random

$$\sum_{(i,j)\in\mathcal{D}} F(i,j) \cdot eq\big((i,j),(r,r')\big) = 0$$

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[LFKN'92]

Linear evaluation

$$\widetilde{W}(\vec{\gamma}) = v$$

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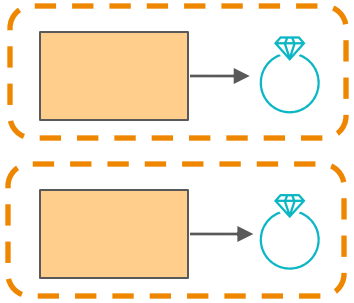
Linear evaluation

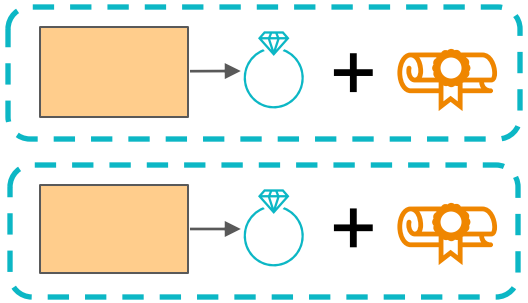
Folding-friendly

Final Scheme

Step 1: Commit



Step 2: Linearize



Final Scheme

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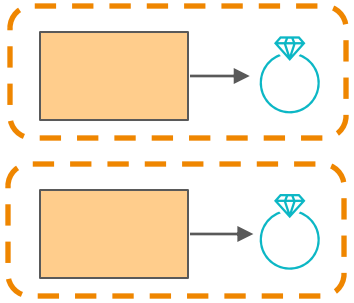
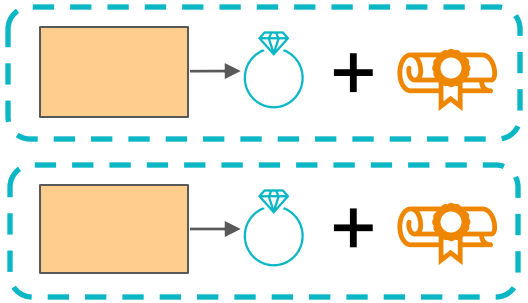
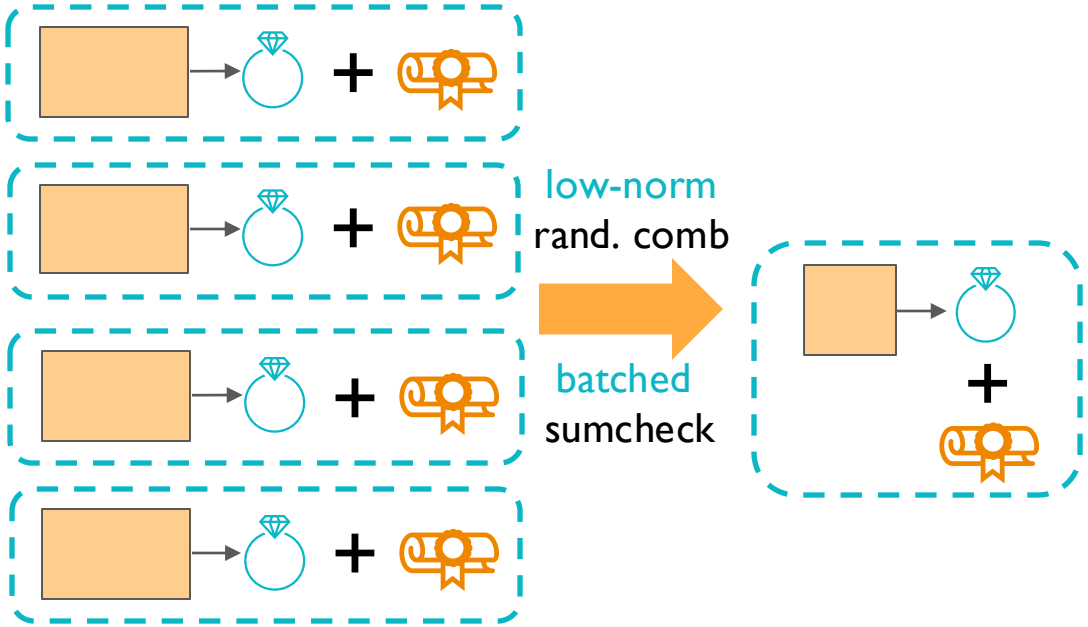


Step 2: Linearize



(Combined) Step 3:
Decomp + Range-chk + Fold



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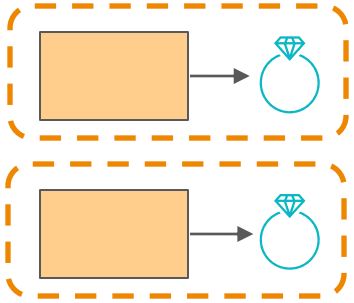

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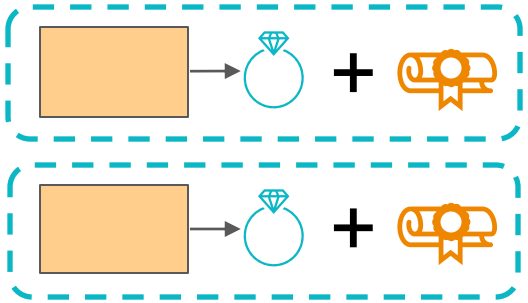

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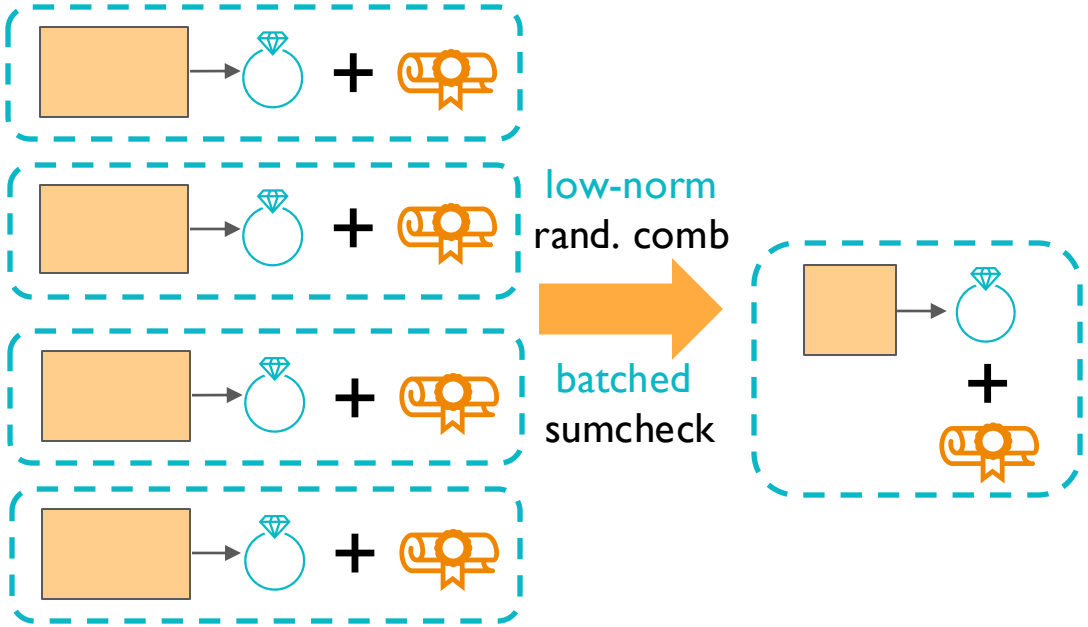

Folding Verifier:

- Batched Sumcheck: $O(\log n)$ Rq ops
- Rand. comb: $O(\lambda)$ field ops

Final Scheme

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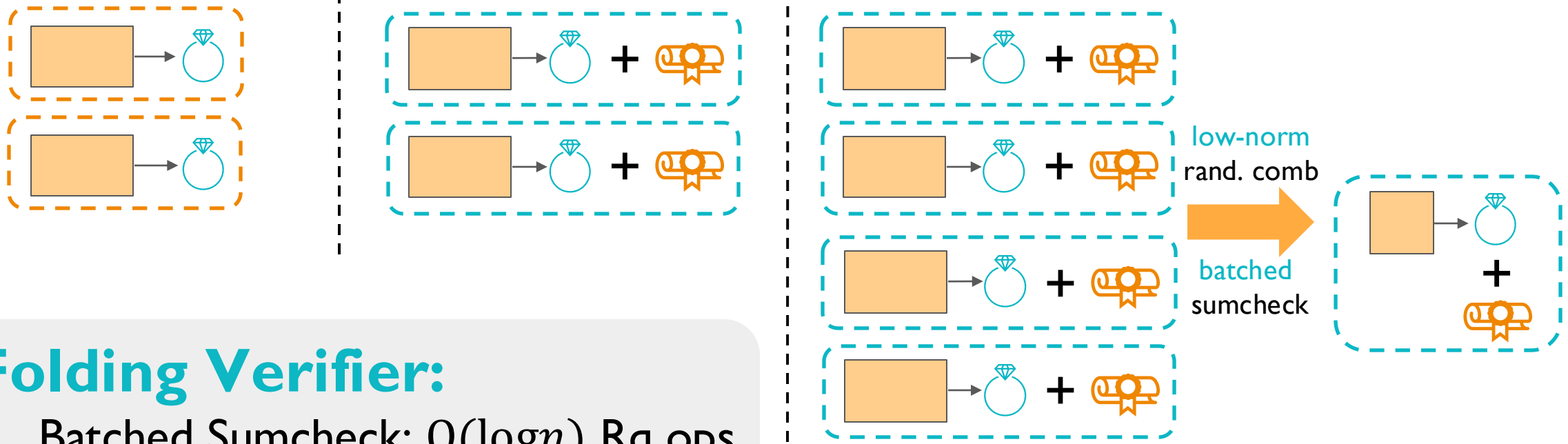

Folding Verifier:

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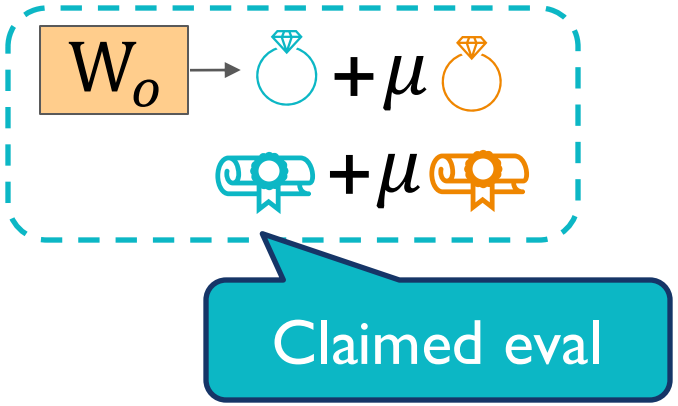
[BC'25, NS'25]: Replace ring w/ field ops

Knowledge Extraction

1st folded instance-witness pair:



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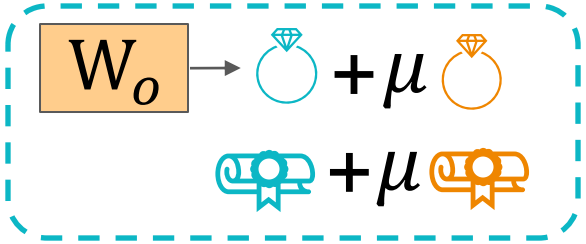
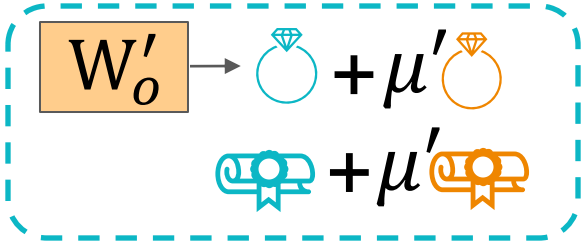


Rewind*

2nd folded instance-witness pair:



*: We actually rewind more than two times.

Knowledge Extraction

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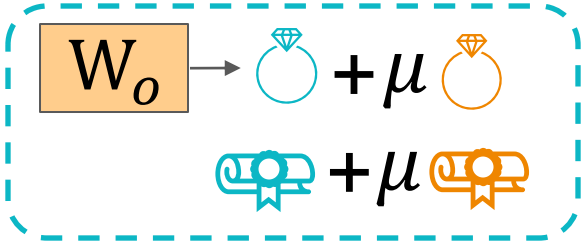
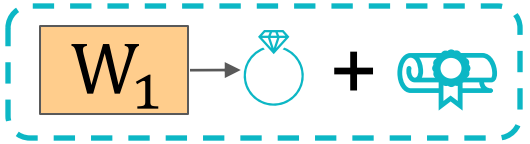
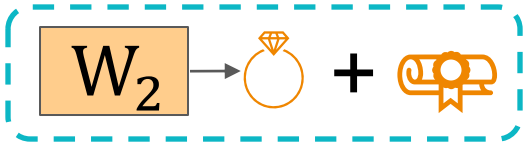


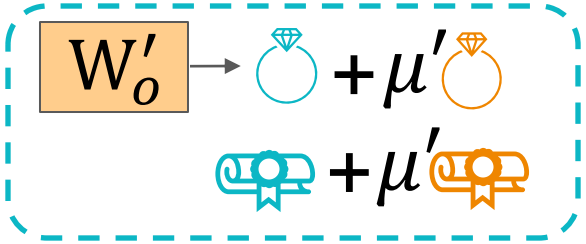
Rewind*

Interpolate





2nd folded instance-witness pair:



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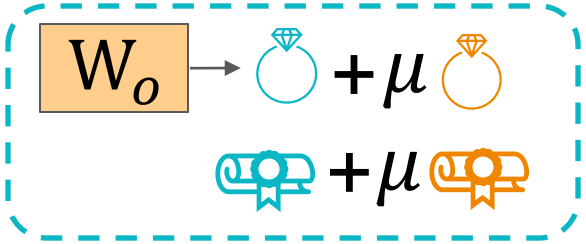
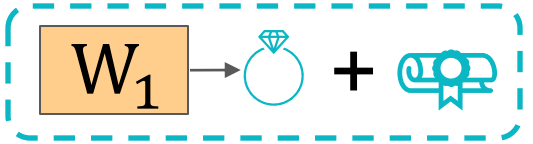


Relaxed binding



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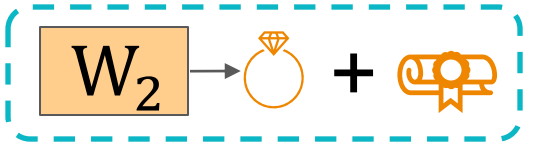
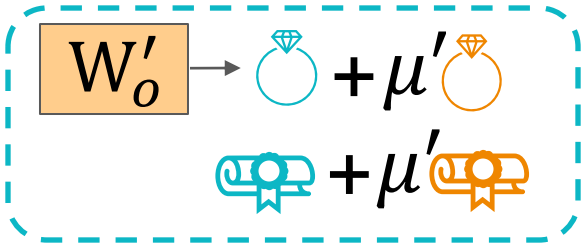


2nd folded instance-witness pair:



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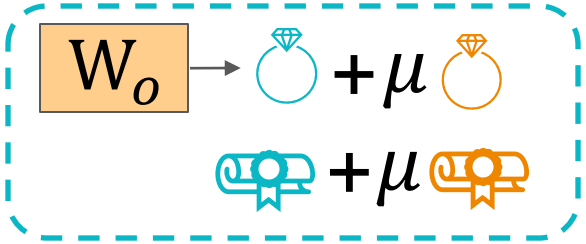

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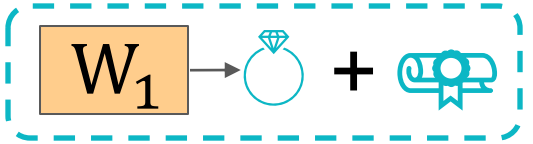
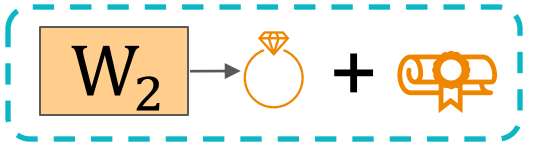
Satisfies eval claim





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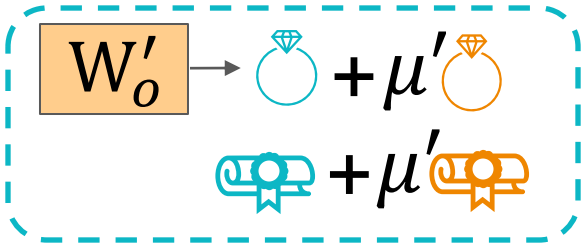

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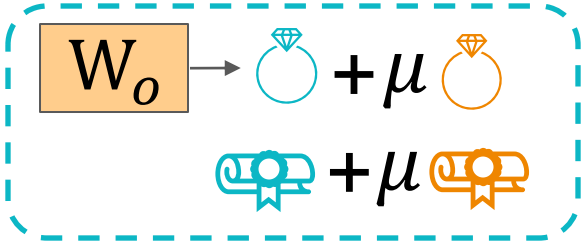

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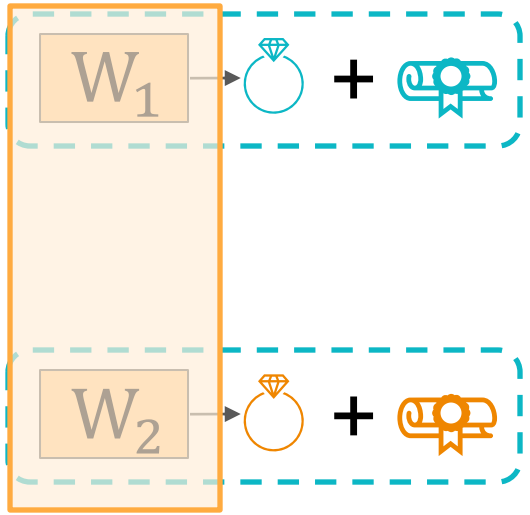

Low-norm by
Sumcheck soundness

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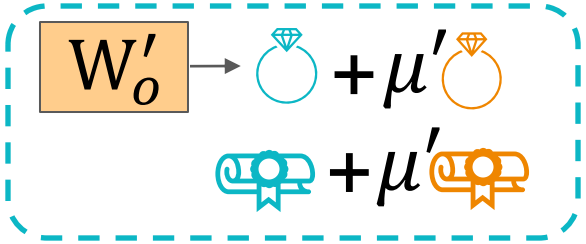

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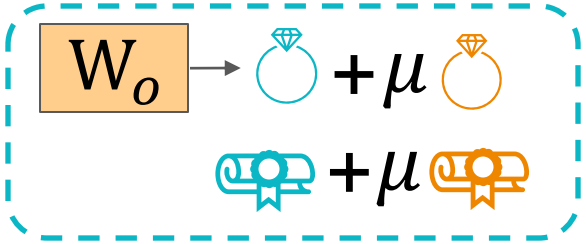

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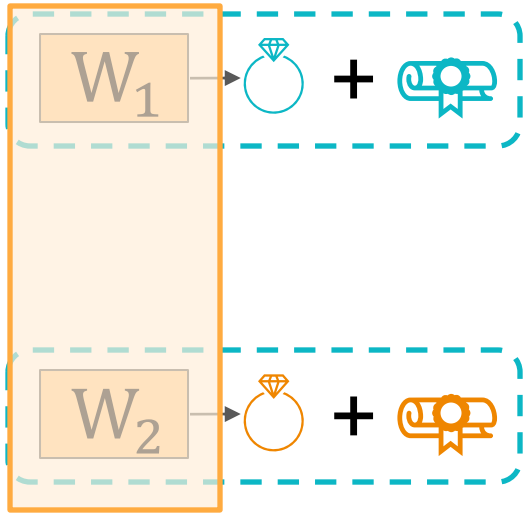

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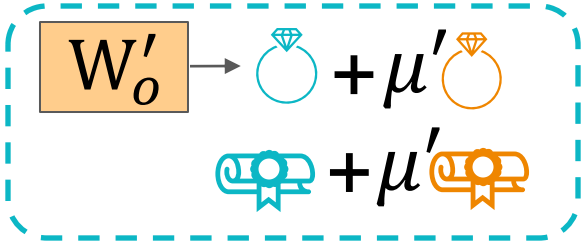

More subtleties on supporting witness vectors over R_q

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Summary & Future Work

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QROM Security Analysis

THANK YOU

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